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# Historical study of school mathematics textbooks at foundation level to the present day 

Doctoral (PhD) Dissertation Thesis by

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## Theme of the dissertation and significance of the theme

Textbook publication has transformed in the past twenty years. The "single textbook" concept, which lasted for several decades, has been replaced by a multiplicity of school textbooks, some which are similar conceptually, while others differ in their professional methodology. Orientation between the textbooks remains a serious challenge for those experts who deal with the theoretic and practical issues of the school textbooks.

In 2006 the Textbook Research Institute (Tankönyvkutató Intézet) made a comparative analysis of school textbooks at senior grade (Fischerné Dárdai-Kojanitz, 2006). However, no vertical or horizontal analysis of mathematics textbooks at junior grade has been attempted so far. In my dissertation I aim to do this.

In my paper I have overviewed the historical development of mathematics textbooks at junior level, starting with the Arithmectica of Debrecen, published in 1557, which might be regarded as the first schoolbook in Hungarian, and continuing to the present day (the 2009-2010 school year), examining school textbooks related to the teaching of computation, arithmetic, later arithmetic-geometry, and then mathematics, the latter which I will subsequently refer to as mathematics schoolbooks. My study focuses on the foundation level of mathematics education, covering the present junior grade classes $1-4$ (and the corresponding cycles used in previous education systems). In this paper, I have used the term "foundation" according to the present National Curriculum.
I have completed detailed analysis of publications of the work of Károly Császár called Computation (1883). More specifically, in the 1880s an international reform movement was launched to renew mathematics education. In 1891 The Mathematics of Physics Society was established in Hungary, which purposed the reform of mathematics education. To monitor the changes taking place during this period I have also analysed the last schoolbook published before this time (in 1883). I have made a vertical investigation from 1883 till 2010 in regard to educational content appearing in mathematics schoolbooks for classes $1-4$. I have made a horizontal investigation of the same issues among schoolbooks studied in the selected period.

The research indicates numerous problematic elements in the textbooks. These disputable issues lend themselves to further research.

Aims of research
I determine the aims of the research as follows:
To overview, trace and analyse the process of the mathematics schoolbooks published for junior classes in the past centuries since 1557, with a particular focus on the period from 1883 to 2010 .

To discover the tendencies in four categories: the construction of curriculum, the selection of exercises, the use of technical terms, and the function of illustrations.

To monitor the appearance of the discovered tendencies in schoolbook families presently in circulation (according to textbook list for 2009-2010 school year).

To provide an overall view on the structure, construction of curriculum, the identities of a selected topic - according to major aspects -, the identities and differences of methodical composition, the roots of differences, contributing to the research of textbooks and to the development of mathematics-methodology.

To provide assistance to education experts- from junior grade schoolteachers up to the editors of schoolbooks - to get oriented in the torrent of textbooks, developing this way the methodological awareness. For the sake of this, to compare the data produced (ordered) by the
market with the 'order of rank' of the schoolbooks analysed and evaluated in my present paper.

To contribute the improvement of the efficiency of the everyday educational practices by developing the methodological culture and awareness.

The results may assist to reconsider that range of topics whether the schoolbooks presently in circulation represent different methodical concepts.

## Research issues

Can any difference or similarity be discovered in the structure of curriculum, in the didactic construction, as well as in the chronologic processing of each curriculum or within the same periods in the schoolbooks from 1557 till 2010
in the curriculum,
in construing questions and exercises,
in learneability of the texts of the schoolbooks, with special regard to the use of technical terms,
in the function of illustration?
In what and how do the recent mathematics schoolbooks rely on the curriculum construction and methodological concepts of the studied mathematics schoolbooks, on the traditions of the mathematics schoolbooks used in previous eras?

Whether the studied textbooks, families of schoolbooks really represent the different concepts, or are there representative ones among them, which are followed by other textbooks without or in lack of individual concepts when elaborating each part of the curriculum?

Is there a demanded methodological awareness in the pedagogues when selecting the schoolbooks; and a sophisticated critical approach, which is required to select the really 'adequate schoolbook' for their students and themselves?

## Structure of the paper

I introduce the scheme of the structure of my paper for the sake of better understanding of the large volume of material to be examined (Figure No. 1.). At first I examine the scientific literature background in connection with the approval of textbooks, with the analysis of schoolbooks (Point 1. on Figure No. 1.). Then I analysed the elementary level mathematics schoolbooks published in Hungary (2.). I divided this part into three sub-categoryies. As a starting point, I have analysed the first six books published in Hungarian, from 1557 till 1777, till making mathematics education obligatory, dealing with the education of the elementary arithmetic, according to four aspects (bibliography attributes, structure of curriculum, didactic solutions, language) (2.1). The intellectual movement of the $18^{\text {th }}$ Century modified the conceptions which formulated on the teaching of arithmetic so far, and started to become uniform by the end of the $19^{\text {th }}$ Century in regard to the teaching of arithmetic. Following the overview of the development of these directions (2.2) I have studied 74 selected schoolbooks of 21 textbook families between 1883 and 2010 according to the four aspects mentioned in Point 1. above (2.3), in five different time periods ((from period 1 till period 5, $\mathrm{k}_{1}-\mathrm{k}_{5}$ ). I have divided each period in two parts: in the first part I briefly overview the education policy relations of the certain era ( $\mathrm{k}_{1}$ - 5. paragraph $a$ ) ), in the second part I analyse the selected schoolbooks ( $\mathrm{k}_{1-5}$.paragraph b)).

In the forecoming chapter (3.) I have studied eight schoolbook families published in the 200910 school year (nine schoolbooks altogether), the second volume of each chapters examine the same topics from three aspects (see under), which study also involves the handbooks of junior grade teachers (guidelines, workbooks, exercise workbooks for junior grade teachers etc.). In the following (4) chapter I have studied the aspects of schoolbook selection of pedagogues on the basis of the data of a questionnaire covering a sample of 450 persons, respectively I summarized the lessons drawn in connection with selecting the textbooks. The last chapter (5.) is the composition of the summary and the outcomes.


Figure No. 1.Structure of the paper

## Selection of research material

I have analysed the first six, mathematics books written in Hungarian between the $16^{\text {th }}$ Century and 1777. I believe a brief investigation of these books is important as their curriculum content, didactical motivations, as it turned out later, formulates the basis of the following elementary level mathematics schoolbooks.

The Ratio Educationis I. made mathematics education compulsory for the second grade of the mother tongue schools in one class per week as an average, then the Ratio Educationis II. made the teaching of arithmetic obligatory for the first grades and define the curriculum content which should be taught. The first mathematics (computation) schoolbooks were published in Hungarian at that time. In the $19^{\text {th }}$ Century the elementary arithmetic curriculum was regarded as almost defined. However, there were passionate debates on the methodology. Four basic principles fought with each other (Beke, 1911).

Formulation of the notion of numbers based on the aspect.
Formulation of the notion of numbers based on the many-sided study of numbers.

Formulation of notion of numbers based on counting and creation of series.
Formulation of notion of numbers based on processing the numerical relations.
I studied the appearance of these methods in the national and international textbooks of the era.

Joint presence of these four basic principles appeared in the schoolbooks by the end of the $19^{\text {th }}$ Century. Therefore I have studied the textbooks for junior grades from 1880 till today. In the period between 1880 and 1945 numerous elementary school textbooks were published in parallel. From 1945 till the end of the years of 1990s - apart from the experimental schoolbooks - one textbook were used for each grades of each school types. The number of schoolbooks were increased, together with the free selection of textbooks. I used the database of OPKM (Hegedős-Tóthpál-Kálmán, 1985) to choose the discussed schoolbooks. I involved schoolbook families into the study, strictly. I decided on schoolbook families, because in the forthcoming studies I can obtain pictures both horizontally (within one grade) and vertically (on the basis of grades following each other) on the construction of the curriculum of each periods and per periods.
By dividing the era arching from the turn of the $19^{\text {th }}$ and $20^{\text {th }}$ Centuries until today into five periods, I selected five schoolbook families as a maximum, which were in use in the majority of the mentioned era. Beyond this, the selection was random.
The $1^{\text {st }}$ era ( $k_{1}$ on the Figure) is the age of the turn of the $19^{\text {th }}-20^{\text {th }}$ Centuries. Following the Compromise (1867) in parallel with the modernization of the elementary school education and the development of paedagogical science, in the years of 1880s a reform movement was initiated internationally to renew the teaching of mathematics. The Hungarian Society of Paedagogy was founded in Hungary in 1891, which started to involve paedagogy into science, similarly to the Mathematics and Physics Society, and the movement of Hungarian mathematics teaching reform was initiated.
The $2^{\text {nd }}$ era $\left(k_{2}\right)$ is the period following the Paris Peace Treaty (1920), when Kuno Klebelsberg minister of religious affairs and public education wished to raise the general level of education by transforming the content of schooling (Pukánszky-Németh, 1996).
The $3^{\text {rd }}$. era $\left(k_{3}\right)$ is the period after World War II., the age of reforming the school system, the period between 1947 and 1951.
The $4^{\text {th }}$ era $\left(k_{4}\right)$ was characterized by the single schoolbooks. I have divided this era into 3 periods. The first period is before the 1962 curriculum, the second is between 1962 and 1978 curriculum, and while the third period is the period from 1978 till 1990 (this was the beginning of the degradation of the single schoolbook concept).
The $5^{\text {th }}$ era $\left(k_{5}\right)$ : from 1978 till the beginning of the years of 1990s when Mathematicsworksheets were in circulation, which were replaced by the family of schoolbooks after a short revision, but maintaining the mathematical, methodological concepts of the worksheets. Which are on the list of textbooks till today. Therefore I compared this family of schoolbooks and the three family of schoolbooks used in the largest volume from the list of textbooks in 2009/10 school year.

I aimed to demonstrate such correlations with the analysis of the four contemporary families of mathematics schoolbooks in the $2^{\text {nd }}$ chapter, which can be read out onlz problematicly from the textbooks merely; therefore I have involved handbooks which are connected to the present schoolbooks into the study. I have selected the second grade schoolbooks from all the junior grade family of schoolbooks from the 2009-10 list of textbooks. Exclusively those chapters, which are connected to the 4 times multiplication table ( $3^{\text {rd }}$ point in the Figure).

In the following paragraph I am attempting to explain why the 4 times multiplication table was selected in the study. The preliminary exploration of the schoolbooks showed that the sequence of teaching of some multiplication tables is different. On the other hand there is a difference in placing the formulation of multiplication notion, as well. There is such schoolbook, which formulates the notion of multiplication prior to the multiplication tables, there is such, which does it in parallel. However, when we study any methodical process, the formulation of the notions connected to multiplication had been completed at teaching the 4 times multiplication table.

## Applied methods

Due to the big time diversion of the books published till Ratio Educationis I. I separately analysed the books according to four categories. First, the bibliographic attributes, such as the features and volumes of creating the book. This was followed by the study of the structure of the curriculum, than I highlighted some presumably interesting didactic processes. Finally I turned to the study of the language of the books, as the Hungarian technical terms started to develop at that time. I am going to study the later schoolbooks according to different approach, due to the significant changes.
( $2^{\text {nd }}$ point in Figure) The preparatory work stages of schoolbook study of the period arching from the turn of the $19^{\text {th }}$ and $20^{\text {th }}$ Centuries till today: sampling, determination of units of analysis, establishing the system of categories and coding the contents. As I had to select large quantity of data, I produced the sample with cluster sampling. I wanted to determine the groups of elements on the basis of curriculum belonging to the given period, reserving the possibility that different groups from these can also be found, even though not too much.
I divided the content analysis into three steps:

1. The stage of coding: I classified certain parts of the text of the schoolbooks to existing categories.
2. The stage of analysis: I processed the coded contents with statistical methods.
3. I have already obtained some information on the latent contents, as the joint occurrence of several codes holds denotational content on the one hand, while on the other hand ideas not being present can have denotational content, too.
The stage of interpretation: Explanation, interpretation and drawing the conclusions on the obtained statistical results.

Beside the analysis of content I made interviews to confirm or to reject the deducted conclusions, the aim of which, beside the confirmation of the quantitative results, to discover newer qualitative data. I used 'professional in-depth interview' (Nádasi, 1996) genre regarding the kind of the interviews, in which the structured interview can be amended with unstructured talks, depending on the messages in connection with the theme.
In order to achieve further results and more shaded approaches I appraised the practices of schools in selecting and using textbooks by interrogation with written questionnaire (electronic).
The comparative study of five periods from the turn of the $19^{\text {th }}$ and $20^{\text {th }}$ Centuries till today covers several aspects, therefore I categorized these aspects. My primary goal was to produce an objective measuring system by establishing the category system, with the help of which I can prepare the working tables inevitable for the analysis. By preliminary reading the schoolbooks I made a survey on which elements would be practical to study. I exploited the
series of aspect by Mrs. Fischerné Dárdai (2008) to determine the categories, and I introduced the following four categories:

1. Summary of lectures ( $s_{1}$ in the Figure),
2. Questions and exercises $\left(s_{2}\right)$,
3. Learnability of pedagogic text limited to the technical terms $\left(s_{3}\right)$,
4. Bibliographic aspects, limited to the illustrations ( $s_{4}$ ).

The comparison of the summary of lectures required the study of the complete series of textbooks implicitly, while it was eligible to compare schoolbook chapters of each topic, when studying the other three categories.

1. Comparison of the summary of lectures was completed in two categories with the aim of exploreing the topics of the schoolbook, to emphasize the identical and different, the vanishing and re-appearing contents.
a) I have analysed the topic on the level of schoolbooks, estimated their volumes, which was measured according to the numbers of pages, then expressed in per cent. Therefore I can get index numbers on the weight of the curriculum within each schoolbook on the one hand, and also, it becomes estimable how important the authors regarded the appearance of the given curriculum in the textbook. On the other hand typographic parameters of the various schoolbooks (type area, type of fonts, spacing, number and size of figures) are different, for comparison purposes the relative comparison is more suitable than the absolute one. As a result of the study it turned out which topics appeared with what impact, and which are vanished.
b) I wished to reveal the concepts of each topics' discussion, the differences and identities of processing the curriculum by detailing their contents, in the topic content.
2. I completed the analysis of Questions and exercises for that purpose to discover:
a) the number of occurrences of the proportion of exercises with number and texts (in order to make the changes of occurrences of these exercises demonstrable),
b) the changes of the types of exercises with numbers,
c) the changes of the types of exercises with texts.

As there is a large quantity of data available for me, I have generated the measurement units by cluster sampling. It was apparent from the preliminary study of the textbooks that the teaching of mathematics for $1-4$ grades considers the foundation of the notion of numbers and operations as the most important in the past 100 years, with new curriculum content from school year to school year. I have endeavoured to involve such topics into the material of study, which can be found in every schoolbooks for the relevant grades (for example crossing ten in first grade), or their lack of characteristic to the given era. I had to select different topic for each grades, namely there is no such topic, which appears in all the schoolbooks, at all the four grades (for example crossing ten is a curriculum in the first grade only). On the basis of all these, I have selected chapters connected to the following topics for the various grades:

Crossing ten for the first grade, but starting with the third age, as in the previous times there was no first grade schoolbook for the families of textbooks.
In the second grade I selected the parts connected with the 4 times multiplication table as a unit for the second grade; whenever it could not be separated in an ample way, I took the parts dealing with increasing the range of numbers from 30 till 40, depending on the dissection of the textbook.
In the third grade I studied the chapter connected to written addition.
In the fourth grade I compared the topics of fractions, in regard to the types and topics of exercises and the types of exercises with numbers and texts.
As a first step I have separated the exercises with numbers and texts. I regarded as exercise with numbers those exercises, in which a given algebraic term or formula shall be solved with
numbers and operators. I relied on the units of the authors when counting the exercises with numbers and I did not count how many operations (item) belong to a certain exercise. I classified the exercises with numbers according to their occurrence, according to the place of the unknown, and I compared their occurrences accordingly.
I have divided the non-number exercises into two groups:
a) Exercises demanding purely subject manipulation - this type was not studied within the framework of the present research - and
b) Other given exercises with texts. Hereinafter I refer to these as exercises with texts.

I have introduced five groups to categorize the exercises with texts:
a) Direct wording exercises solvable with a simple, single operation - meaning, there is a direct reference in the text to the operation to be carried out,
b) Indirect wording exercises solvable with a simple, single operation - there is no direct reference in the text to the operation to be carried out,
c) Complex exercises - in which conversion of measurement units is required,
d) Complex exercises - which can be solved with many operations,
e) Complex exercises - which contain unnecessary data.

There may be such exercises among the complex exercises, which could be classified into several groups. Therefore it should not equal the number of all exercises with the sum of the number of exercises in the above mentioned groups by all means.
I have drawn a conclusion to the latent content and the changes of the contents by studying the content of exercises with texts, on the basis of the topics of the exercises. The characteristic content of exercises with texts refers to the age, to the concept of the autor's view of a child, to his own view, to his opinion of value etc.
3. The study of learnability of pedagogic texts is determined by the aspect we want to claim. We can find surveys in the scientific literature for example on the basis of expectable values, on the basis of the expectable functions, on the basis of the nature of the texts, on the basis on the grammatical structure of the texts (Kojanitz, 2004).

Geiling studied the understandability of pedagogic texts on three levels, on the levels of words, sentences and the text (id. Kojanitz, 2004; Geiling, 1987).
Foreign words and technical terms received special attention on the level of words during the analysis of the pedagogic texts when analysing the schoolbooks. According to Kojanitz (2004) most of the understanding and learning problems are caused by technical terms. The occurrence of technical terms determines the understandability and the learnability of the text to a large extent. However, it also shows how important the authors regard to get the technical terms acquired and learnt in a given chapter.

The mathematical texts show significant differences (on all three levels) from other, literary or scientific texts. The use of large number of symbols - we can call it the level of words -, and the combinations of numbers - level of sentences - is characteristic in the expression of mathematical contents. The use of these symbols mostly unified all over the world; it carries the same content for every people dealing with mathematics, independently from the natural language of the mathematical text (Czékmán, 2010). I have experienced by the preliminary study of schoolbooks that the use of symbols shows insignificant variance therefore I have ignored their systematic study. I marked in some cases if the use of symbols deviates from the generally conventional.
I wished to make the quantity and the differences of the technical terms measurable, then to compare them with qualitative means, when comparing the schoolbooks of each era.

I regarded those words as technical terms, which are not used in the everyday language, or their common mathematical meaning differs from the everyday meanings. In the selected topics I have collected the technical terms of a given part into a table, this way I have obtained not only the number of technical terms, but the proportion of technical terms and the pages together with the list of vanished, surviving and the recently appearing words. (I did not study the frequency of appearance of the technical terms.)
4. The bibliographic aspects cover the illustrations, as the illustrations play an important role in understanding and memorizing the acquired knowledge. According to the dual coding theory of Paivio (1971) two information coding systems of ours operates during processing the information. We can process the visual, the concrete picture-like information with one of them, while we process the verbal, the abstract information with the other. The different coding is carried out with the aid of our cerebral hemispheres, the connection of which is provided by an 'information bridge', the callosum (corpus callosum). I did not intend to provide any typographic analysis.

I wished to unfold the quantities, the types of the information and their harmony with the mathematical content in three groups of views.

In the first group of aspects I have examined the proportion of the pages and illustrations on one hand, and the proportion of the graphic and textual elements on the level of schoolbooks. Accordingly I classified the illustration into four groups:
a) There is no textual element in the illustration,
b) The textual element is less than 50 per cent,
c) The textual elements are not less than 50 per cent,
d) Illustration with text (for example table filled up with numbers belong here).

In the 2 nd group of aspects I have studied the relation of the mathematical content and the illustrations in the view of problem solving on the basis of Zrinszky (2003) I classified the illustrations into four categories:
a) The category of the directed observation (I classified here not only the observations directed by explanations, just like Zrinszky, but every directed activities),
b) Explanations depending on illustrations,
c) Aspect justification or posterior illustration,
d) Motivating illustration.

In the $3^{\text {rd }}$ group of aspects I classified the illustration into four categories:
a) connected to active transactions, that is the illustration requires activities accompanied with actions.
b) passive admission, preliminary adventures, reviving experiences,
c) the illustration directs the learning process, for example it directs learning activities with exemplary conduct,
d) the illustration is independent from the mathematical content.

I have compared the schoolbooks on the basis of pre-made textbook-profiles with the aid of categories, sub-categories, in break down according to the categories.

On the basis of the four categories and sub-categories at first I studied the various schoolbooks per volumes within each period, then I monitored the changes within each schoolbook family, and finally I analysed the changes between the different eras.
(Figure 3. point) In order find our whether the chapters connected to the 4 time multiplication table in the schoolbook family of junior grades from the list of 2009-10 show any essential difference in the teaching of the multiplication table, and whether there is any precedent in the
previously published textbooks, and if yes, then where these are originated from (Figure Point 2.) I took the above introduced system of categories for basis.

In the category comparison of the curriculum the first sub-category, the range of topics has already been given. In the part dealing with the content of the range of topics, just like till now, I wished to discover the methodological structure of the multiplication table. For the sake of this I studied the placing of the multiplication tables in the yearly curriculum; the introduction of the connected notions: multiplication, subsumption, division into parts, division and division with residue, the interpretation of multiplication, the denominations of the notions used in multiplication.
In the category of questions and exercises besides the proportion of occurrences and the types of exercises with numbers and texts I also studied the mathematical notions closely connected to multiplication in the exercises, as well as appearing in the concentration of the subject.
I extended the category of learnability of pedagogical texts. I carried out the linguistic examination of the 2nd grade schoolbooks appearing in textbook list of 2009-10 in four highlighted areas:

1. Construction, extent and structure of sentences.
2. Accuracy, congruence of the composition of the mathematical content.
3. Lexical study covering three groups of words, taking the appearance of the vocabulary and the technical terms in consideration.
4. The examination of orthography and orthology. (Köves-Szegfü, 2011)

The importance of the study was confirmed by the fact that both the development of cognitive and communicative abilities in teaching of junior grade mathematics is a task, as it is pronounced in National Curriculum (NAT) and curriculum frameworks of 2012. In the mother tongue and mathematical competences reference to the interaction of the language and thinking, the importance, the recognition of cause-and-effect relations, the requirement of highlighting the essence, as well as the understanding of the listened and read texts, the development of text composition, the importance of argumentation, the expansion of vocabulary, the acquisition of technical terms etc. appears as joint elements. The language usage of the schoolbooks equally serves as examples for the teachers and pupils, therefore I think it is important to understandably and exactly phrase the mathematical contents, the questions and answers to validate the educated every-day and scientific language together.

Similarly to the previous chapters, here I have also studied the latent contents appearing in the exercises.

In the bibliographic sub-category I have dealt with the illustrations only. During the reviewed century more and more illustration were inserted into the schoolbooks. The studied textbooks of the present the illustrations constitute fifty per cent of the schoolbooks as an average; and they are closely connected to the exercises. Therefore I have examined the illustrations from the aspect of their connection with the mathematical content. I have grouped the illustrations according to the nature of their connection to the exercises. I determined several categories (and further sub-categories within) on five levels.

1. The illustration is connected to the solution of the exercise.
2. The illustration is connected to the interpretation of the exercise.
3. Calling image
4. There is no connection between the illustration and the interpretation or the solution of the exercise
I have carried out the schoolbook investigations by the method of comparative, qualitative and quantitative analyses. I discovered the occurred changes on the basis of the quantitative data. I have determined the measuring in pages (given by the page numbers of the topic), in
numbers of items (for example: how many exercises are in a unit?), in the frequency of occurrence, I have also expressed the percentile distributions, as well. I used tables, bar charts, spider web charts to visualise the data. This latter was selected, because its structure is suitable to visualise the data of a schoolbook from several views, while the same chart is suitable to compare several schoolbooks from one point of view, just like to compare several schoolbooks from several views. On the basis of predetermined categories I compared the mathematical curriculum by content analysis.

## Result of research

1. Can any difference or concordance be discovered in the structure of curriculum of the schoolbook from 1557 till 2010, in their didactic construction, as well as in processing a curriculum of the same period and chronology?

- in the thematic,
- in the compilation of questions and exercises,
- in the learnability of pedagogic texts, with special regard to the usage of the technical terms,
- in the function of the illustrations.


## Changes occurred in the curriculum

The elementary level mathematics teaching of the past 125 years investigated in details was homogenous in wanting to teach the notion of cardinal numbers as the most prevailing way together with the gradual increase of the range of numbers and with teaching the mental and written arithmetic in the first four school years.

## Gradual increase of the range of numbers on the set of cardinal numbers

All of the first mathematics books in Hungarian introduced the range of numbers to be taught from 1 till one hundred million generally in one step, right at the beginning of the book. In this period the development of number and operation notions were completely separated. With the introduction of the compulsory education the schoolbooks were written for a determined age-group, even at the time of the two Ratio Educationis, which could be a motive of the gradual introduction of the range of numbers later, which brought along the gradual introduction of teaching the operations.
At the time of Ratio Educationis II. In the first grade, children counted till 100, then till 20 or 30 with a few exceptions, in the second grade they counted till 100 , while in the third grade till 1000 , before till one million in the fourth grade, while in the present they schoolbooks they count till one hundred thousand. There are two directions which appear to increase the range of numbers for each grades. One of them increases the range of numbers in one step, and teaches the four fundamental arithmetical operations separated from each other. While the other (after Busse) gradually increases the range of numbers, and in this case the development of number and operation notions are carried out in parallel.

The development of the number and operation notions were carried out in parallel in the second grade in the first two eras, but there were differences in the steps of increasing the range of numbers. From the view of concept-formation this curriculum construction was very effective, because the analogue operations were practiced in small range of numbers, this way they could advance in larger steps later. Out of the three schoolbooks of the third era, one, a schoolbook of Budapest returned to the traditions of the 18th Century. It increases the range of numbers in one step from 20 till 100 and teaches the four fundamental arithmetical operations separated from each other. At the beginning of the fourth era the big 0-1
schoolbooks intended to return to the established usage existing till the mid-twenties Century, then from 1962 the Csáki-Géczy1 schoolbook followed the methodological structure of the Budapest textbook of 1947. Seemingly it was simple copying, or stepping back, but if we also consider the preliminary knowledge of the beginning of the second grade, the methodological mistake seems to be more appropriate for the contemporary eyes. Namely, the preliminary events of the schoolbook of Budapest were the established orientation within range of number of 100 , while it was only range of numbers of 20 in Csáki-Géczy1-2. this methodological practice was followed by the Apáczai and Mozaik schoolbooks selected from the present textbooks, while the schoolbooks of NTK_C and Műszaki increase the range of numbers in three (20-30-100) and five steps (20-30-40-100-200).

In the third grade curriculum, from the years of 1920s till the beginning of the 1950s, the range of numbers were gradually increased to one thousand, while in the schoolbooks of the other studied periods the range of numbers were increased in one step. In the fourth grade in 1947 the range of numbers was increase to one million in one step. The Baranyai schoolbook of 1947 stiffly, similarly to the second grade, increased the range of numbers in steps even above one million. The later schoolbooks generally increased the range of numbers in two steps; however, do contemporary books do this till one hundred thousand.

## Teaching of the four fundamental arithmetical operations, the formulation of the operation notion

The first arithmetic books in Hungarian took to teach the written calculation as their task only. Education of mental arithmetic was not included in the books. In this period interpretation of numbers and teaching of mathematics operations was clearly separated. From 1869 the four fundamental mathematics operations were supposed to verbally know in the first grade in the range of numbers of 20 or 30 , from 1905 only in the range of numbers of 20 . Only verbal additions and subtractions had to be known in periods $3-5$ in the same range of numbers. There is only one exception: 'Our first book' from 1951, which introduced multiplications and divisions in the range of numbers of 20 . All schoolbooks uniformly teach the four fundamental mathematical operations in the second grade, verbally. Methodology was unified by the 20 s . Each of the ranges of numbers increased in steps of ten, the four fundamental mathematical operations were practiced by mental arithmetic, the by written exercises with numbers and texts. Schoolbooks of 1947 did not rely on the previous experiences, they were written in the sign of renewal; differences can be seen in the construction of the structure, in their vocabulary, in almost every momentum, Then the Nagy0-1 schoolbooks returned to the methodology preceding the methodology of 1947, but Csáki-Géczy1 of 1962, then its version the Csáki-Géczy2 schoolbook used the Budapest textbook of 1947 as its basis. It separated all the four fundamental mathematical operations, the multiplication tables were taught in groups, which were possible, because the range of numbers was increased in one step. As a result of this the characteristics of the operations were fixed in the pupils not via its natural way, but they had to deal with their familiarization separately. Apáczai and Mozaik schoolbooks among the present textbooks followed the practice of Csáki-Géczy2. The Apáczai practices the operations separately from each other, and then it tries to interpret the connections between the operations in the part of miscellaneous exercises. The got addition and subtraction practiced in the first half of the school year, while multiplication and division in the second. The NTK_C also separates addition-subtraction and multiplication-division into two half year periods, but increased the range of numbers in 20-30-100 steps. The schoolbook of Műszaki partially returns to the methodology of the turn of the $19^{\text {th }}-$ and $20^{\text {th }}$ Century: it increases the range of numbers of 20-30-40-100 steps, it gets the four fundamental arithmetic operations practiced at each steps.

The major part of third and fourth grades is run by the learning of the four fundamental arithmetic operations in the learnt range of numbers. The Written arithmetic operations gradually get into the curriculum of the third grade from the beginning of the $20^{\text {th }}$ Century, then, they were gradually left out from it by the end of the century.
In the Császár schoolbook for the third grade multiplication and division is not included among the written arithmetic operations, then it gets into the curriculum with single digit and multi-digit numbers, The Beke schoolbook is the first - among the studied textbooks - which handles multiplications and divisions with single and multi-digit numbers as a separate topic. From the schoolbook of 1947 this part of the curriculum was completely omitted, which meant a huge loss of the curriculum, however the written multiplication with single and multidigit numbers was taken back in the years of 1950s, just like the division with single digit divider. The schoolbooks of the present days in their curriculum arrangement and the schoolbook of the Müszaki and Mozaik maintained the written division with multi digit dividers for this grade. Written multiplication with multi digit multiplier and written division with multi digit divider was completely transferred to the next grades. I could not find methodological explanation to the reduction of the curriculum from 1947. The reduction of the curriculum in the contemporary schoolbooks can be justified by the broadening of the topics in the curriculum, and the reduction of the weekly number of classes.

## Teaching of common and decimal fractions ${ }^{1}$

According to the curriculum of the elementary schools of the Danubian Reformed Church District in the years of 1871 and 1880 it included the fractions and the arithmetic operations with fractions. According to Lajos Deső curriculum in 1895 - written for the elementary schools of East of the Tisza River (Tiszáninneni) Reformed Church District - in the first and second grades the derivation of the fractions, in the third grade the arithmetic operations with fractions, while in the fourth grade the decimal fractions were the curriculum, which appeared in the schoolbook of Schultz in 1899. In the era common fractions were not included in other schoolbooks. Two schoolbooks dealt with decimal fractions in the fourth grade, both of the added up, subtracted the decimal fraction, multiplied and divided the decimal fraction with positive integers and positive decimal fractions. According to the curriculum of 1905 the derivation of fraction was a curriculum for the fourth grade. At the turn of the $19^{\text {th }}$ and $20^{\text {th }}$ Centuries the schoolbooks for the fourth grade dealt with the common fractions in their one fourth parts as an average. During the following periods this proportion was decreased, from the beginning of the years of 1960s till the curriculum for 1978 it was even omitted. The present schoolbooks for the fourth grade keep on teaching the fractions began in the third grade, however they do not reach even the fifty per cent of the volume of the first period.

## Teaching of geometric knowledge

Maróthi wrote in the introduction of his Arithmetic that 'Arithmetic is very useful to sharpen the mind of the children, and so does Geometry' (Maróthi, 1743). Even at the time of Maróthi the topics of the measures, the conversions of the measures was regarded as a part of mathematics. At the time of Ratio Educationis I-II geometrics became a non-mandatory subject in the schools of the cities. At that time only that was taught from the geometry, the application and the resolution of which occurred in the everyday life. (Ratio Educationis I).
From the turn of the 19th and 20th Centuries the topics of the measures and conversion of measures were omitted from the textbooks for the first grade only in the period between 1954 and 63. The topics of geometric knowledge in the first grade appeared in the Our first book in 1951, then in every schoolbooks from the curriculum of 1978 continuously, with the

[^0]exceptions of the Apáczai textbook. It appears at the senior grades, disregarding the few exceptions, since the curriculum of 1978, especially the geometric exercises were grouped around the topics of geometrical solids and figures, and around transformations.

## Changes occurred in the issues and tasks

I began the study of questions and exercises by determining the number of exercises of the selected topics for each grade. The number of exercises in the contemporary schoolbooks from the first grade till the fourth decreases compared to the turn of the $19^{\text {th }}-20^{\text {th }}$ Centuries. However considerable conclusions cannot be deducted merely from the number of exercises, because there are different item quantities belong to each number of exercises in the various schoolbooks.

Exercises with numbers. In the first grade the numbers of exercises were decreasing in a great extent as the time went on, but the trend line of the number of types of the number exercises shows just a little move. In the second grade the number of exercises and variability of the exercises connected to the range of numbers of 40 , and the 4 times multiplication table. A great difference appears between the types of number exercises of the schoolbooks of 1-2 and 3-5 periods (Diagram 1), the most significant reason of which is the change of the strategy of the increase of the range of numbers. In the studied topic of the third grade the trend line of the exercises shows a little decrease. The number of exercises connected to the teaching of fractions and decimal fractions in the fourth grade shows the greatest decrease. It reaches its minimum at the time of the single schoolbook period; however it can not reach the level of the turn of the $19^{\text {th }}-20^{\text {th }}$ Centuries even today. One of the reasons of the decrease of the exercise type numbers is the decrease of the content of the curriculum, which is due to the change of the aspect in the method of increasing the range of numbers. The emphasis was placed on the operations with fractions when teaching the fractions in the first two periods, while it is rather on the formulation of the ideas today.
2. era

5. era


Diagram No. 1.
Appearance of the types of exercises with numbers, second grade

Exercises with texts. Trend line of percentage proportion of the exercises with texts and the percentage distribution of the types of the exercises from the first till the fourth grade between 1883 and 2010 is the following:
Exercises with texts occur in the studied chapters of schoolbook for the first grades in the third-fourth period in an ever decreasing fashion. As an effect of the new mathematical exercises with texts occurs in bigger proportion in NTK_C textbooks, but the other contemporary schoolbooks do not follow this tendency. The minimum number of exercises with texts is shown by the present Mozaik schoolbook. About the type of the exercises with
texts it can be determined that the texts of the studied exercises became simple with time. There are more and more simple exercises with direct text. In the examined chapters of the present day Apáczai and Mozaik schoolbooks only these types of exercises can be found, however in the NTK_C and Müszaki schoolbook there are complex exercises in a relatively larger extent.
In the second grade in the $1-2$ periods there are more exercises with texts occur in a growing extend. A decline can be seen in the 3-4 period; the schoolbooks of NTK_C and Műszaki exceeds, while the schoolbook of Mozaik reaches the proportion of the $2^{\text {nd }}$ period among the studied contemporary schoolbooks. The amount of complex exercises solvable with multi arithmetic operations is also shrinking.

In the topic of verbal and written addition in the third grade the proportion of exercises with texts was increased till 1947, beside the simple exercises with direct text complex exercises also appear. The proportion of the exercises with texts is decreasing from this time. Exercises with direct texts only can be found in several schoolbooks (Diagram 2).


Diagram No. 2.
Trend line and proportion of the types of the exercises with texts, third grade, 1883-2010.

In the topic of fractions in the fourth grade the trend line of the exercises with texts shows the biggest decrease. In the contemporary schoolbooks the proportion of the exercises with texts is far less than it was at the beginning of the $19^{\text {th }}-20^{\text {th }}$ Centuries. In connection with the education of the topic the subject manipulation gradually becomes general, starting with the Gaál textbook. This also shows the change of the approach. In junior grades instead of the arithmetic operations with fractions the emphasis is replaced to the formulation of the notion fractions.

The question may be arisen whether the significant decrease of the number of the exercises with texts and the complexity of the texts could make unfavourable effect on the cognitive abilities, like the ability of reading, accentuation of essence, systematization etc.

## Changes in the use of technical terms

The technical vocabulary of mathematics is special compared to other technical vocabularies. Its usage is broader than most of the technical vocabularies, because in its elements or in its totality it is taken over by other disciplines, and mathematical terms are also required in other areas of the everyday life. Therefore it must be organically integrated into the curriculum of the schools. The mathematical vocabulary contains many symbols, signs, which can express codes or relations between terms; by their usage communication will be clear.
By studying the appearance of the technical terms it can be determined that in the first grade from the years of 1960s the technical terms almost disappears from the studied chapters (possibility to avoid the 'accusation of verbalism' at that time), which are returning in the contemporary schoolbooks in a growing number. In the second grade in the selected topic
bigger divergence is shown within and between the periods, but the trend line moved around new technical terms with small fluctuation. In the third grade at the turns of the 19th and 20th and the 20th and 21st Centuries the average of the technical terms per pages equals, while there were less technical terms at the mid of the century. The usage of technical terms in the fourth grade was less characteristic in the topic of fractions. Nowadays increase is shown, the number of technical terms per pages are more and more, but their average does not reach even the half of the average at the turn of the 19th and 20th Centuries.

## Changes occurred in the appearance and the function of illustrations

Study of occurrence of the illustrations. Illustrations appear at every second-third pages maximum at each grade till 1947, but it can also happen that the chapter does not contain illustration at all. Together with the development of the printing techniques the quality of the schoolbooks as a printing product and the average number of illustration per pages spectacularly increased to the turn of the 20th and 21st Centuries.

Relation of the mathematical content and illustrations. Illustrations occur in the first grade in the fourth period in the largest number, but generally these are not connected to the mathematical content, does not assist the mathematical thinking. The printing technique enabled the appearance of four colour-print schoolbooks in this period; however the authors of the textbooks generally did not consider this technical advantage to be exploited for the sake of learning. The most appropriate description of the illustrations may be that they were distractive pictures however the real goal was probably the motivation.
In the second grade at the beginning of the $20^{\text {th }}$ Century the illustration served the aim of the passive recall of knowledge then they were gradually replaced by illustrations connected to active work. In the 20 s illustration directing attention appears then gradually their extent is getting to be increased. This can reach 7 per cent in an average in the case of the contemporary schoolbooks.
At the beginning of the $20^{\text {th }}$ Century in the chapter of written addition in the third grades maximum one illustration can be found. In the years of 1960-70s the motivating illustration became characteristic, and then they were replaced the explaining illustration relying on visualisation in the schoolbooks of the present days. The proportion of illustrations serving reviving the passive knowledge connected to the active work and directing the learning process started to be compensated.

The change of the aspect in teaching of the fractions in the fourth grade can also be seen in the illustrations. While at the turn of the $19^{\text {th }}$ and $20^{\text {th }}$ Centuries explanations relying on demonstration and the justification of aspects was the most characteristic, at the turn of the $20^{\text {th }}$ and the $21^{\text {st }}$ Centuries the large proportion of illustrations directs the observations. The majority of the illustrations are connected to active work. At the teaching of fractions in the fourth grade in the studied overviewing 128 years there was only one schoolbook, the present day Apáczai, the illustration of which are independent from the mathematical content.
Consequently it can generally be observed that the authors exploit the benefits of the new technology in a less and less autotelic way, according to their intentions the illustrations serve the education more and more.
2. How and in what extent do the mathematical schoolbooks of our days relay on the curriculum structural, methodological considerations, the traditions of the analysed mathematics schoolbooks in the study used in previous times?

I studied the teaching of the multiplication table in chapter 3 in the school year of 2009.10 the nine mathematics schoolbooks from the textbook list, in the volume for second grades from the view of the curriculum content, methodology, language and illustration - and task relations.

Concepts connected to the teaching of the multiplication table are approached by the present day textbook in a much divers form, however examples of each and every approach can be found in some mathematics books from the $18^{\text {th }}-20^{\text {th }}$ Centuries. Every second grade schoolbook starts from a nearly identical mathematical base at the end of the first grade, but the special output requirement determined by the framework curriculum was substantially different.

Localization of teaching the multiplication tables in the annual curriculum shows three different tendencies. Also three variations can be separated in the teaching sequence of the multiplication table. Everybody demands the interpretation of multiplication and divison in the area of opeational notions, however they do not mean the same thing under these notions. Neither the denomination or the elements, nor the read out of multiplication, nor the interpretation of division is uniform, not to mention their markings. There are five options in the nine shcoolbooks for the connection sequence of teaching multiplication, division partition and remainder division.
The numbers of exercises to be solved in this topic, as well as the average numbers of exercises being solved within one lesson are significantly different. Two schoolbooks can be selected in such a way that the average numbers of exercises of the lesson are four times more in one textbook than in the other. This proportion shows that the authors of the schoolbooks regard the necessary numbers of exercises for ample practicing very differently (probably in the absence of basic research), but there are indications that the authors of the textbooks give a chance for the elementary school teachers to differentiate.

Concentration within the subject, - which really became prevailing with the appearance of the new mathematics - is not applied in the studied chapter of the four schoolbook at all.

The numbers and proportions of exercises with texts also show great divergence. Two schoolbooks can be selected in such a way that there are nine time more exercises with texts in one of the schoolbooks in a given topic than in the other. Great differences are also shown in the appearance of the types of exercises with numbers.
Language feature of the the studied schoolbooks is the great quantity of pendent sentences on the level of sentences. As a consequence of the unjustified structural and substantual simplification the sentences often can neither meet the demand of unambiguous, exquisite composition, nor the precise mathematical composition. The use of unexposed code (tie together!, Continue!, Correct etc.) is originated from the complex mathematics and spread around the printed student handbooks for 1-4 grades. Arithmetic schoolbooks published at the beginning of the $20^{\text {th }}$ Century paid greater attention on the education of the mother tongue than the present day ones.

More varied notion vocabulary appeared in significantly, more group of topics on the level of words than in the conemporary ones. Comparing the appearance of non-mathematical contents in the schoolbooks at the turns on the $19^{\text {th }}-20^{\text {th }}$ and $20^{\text {th }}-21^{\text {st }}$ Centuries, the decrease can be seen (Digaram No. 3). In contradistintition to the studied chapters of the schoolbooks of the first era there is no reference to moral features - like trustworthiness, helpfullness, patriotism - to the respect of property, to the protection of the environment, to the hygienic standards, to professions. It can be seen on the diagram that even those elementary topics, like food is hardly appearing in the texts. Presently the notion of the complete familiy appears
only in one schoolbook, altogether in two exercises. The one-parent family, the relation of the mother, the child, the grandmother and grandchildren appears a little bit more. I was not able to find any reference to the companion from the contemporary age-group, to any realtion with companions in the studied details of the present day schoolbooks. It can be stated that a significant part of learning options remains unexploited in this regard.


Diagram No. 3.
Topics and the numbers of the connected notions in the chapters connected to the 4-times multiplication table of the second grade schoolbooks of the first and fifth eras

The majority of the illustrations are now connected to the interpretation, to the solutions of the exercises.

Differences of curriculum perception, process make the passage difficult between the schools teaching from different schoolbooks. However, at the same time, the various terminology, the different interpretations, the methodological knowledge of the teachers can broaden the more efficient learning of the curriculum, in principle.

The question may be arisen whether the teachers are aware of the differences between the various schoolbooks. Many sign shows that they are not, or not sufficiently at least. Ferenc Lénárd primarily sees the problem of the too many schoolbooks, the too frequent exchnage of the schoolbooks in the pedagogues having no occasion, not enough time to get acquainted with the thinking of the students, their perception problems, the faulty interpretations, the frequently occuring stereotypic errors, the impressions of the schoolbooks made on the students in connection with the material of knowledge of the new textbooks. According to him to experience all these would require at least ten years (Lénárd, 1986).
3. Whether the studied schoolbooks, families of schoolbooks really represent different concepts, or there are competent ones among them, which are followed by other schoolbooks without any individual concept when processing each parts of the curriculum?

By investigating the mathematical content in the selected chapters of the second grade volumes of the eight mathematics schoolbook families we can determine that conceptionally two main directions can be distinguished in the compilation of the curriculum, which are most pronouncedly represented by the schoolbooks of NTK_C and Müszaki in regard of the concentration of the subject and the variability of the types of the exercises with texts and numbers. These two schoolbooks consequently stuck to the mathematical concept they represent.

There is no ground to decide whether which of the two pronounced directions will turn out to be more successful, as no competency study is connected to any of the schoolbooks; however it is also possible that no such decision is needed. The other seven schoolbooks take parts over from this or that concept in smaller or greater extent, the consequence of which is that they get in conflict with their own strategy of curriculum compilation and methodology here and there.

We must settle upon the many schoolbooks in the dilemma of single versus many schoolbooks, schoolbooks providing real learning concepts have reason for justification. It may be an optical illusion that our mathematics teaching in the junior grades reached its nadir in the period of the single schoolbook, because of the given schoolbook. When the selectable parallel schoolbooks are on similar low standards, the eligibility will not boost the efficiency of education. I do not think it is right if the single schoolbook would be re-entered, neither the publication of textbooks showing no individual conceptual specialities at the same time.
4. Is there ample methodological awareness in the selection of schoolbooks by the pedagogues, just like the fully developed critical way of looking at things, which is required to select the really 'appropriate schoolbook' for their students and themselves?
The introduced study of the schoolbooks proved that the selection of schoolbooks by the elementary school teachers is not in harmony with the results of my research. Schoolbooks judged as the most appropriate on the basis of my study were not ordered in the greatest number of copies by the schools according to the answers given to the questionnaire, however the schoolbook ordered in the greatest number of copies was among the last ones according to the aspects of the study.
Data prove that schoolbook selection of the schools and the teachers is neither in harmony with the 'goodness' of the aspects of the study, nor with the summarized results of the aspects. On the basis of the schoolbook orders the rank of the first three textbooks - which were the 85 per cent of the total order of schoolbooks - is just the opposite with the rank determined on the basis of my study.

The satisfaction of elementary school teachers is well shown by the schoolbooks, that is whether they use regularly any other complementary handbooks in their classes. More the fourth of the teachers use schoolbooks different from the ordered ones. Users of the schoolbook ordered in the greatest number of copies - Apáczai - 14 per cent characteristically also use the textbooks of other schoolbook families; those who ordered the textbooks of Mozaik and Műszaki, 9 per cent use different schoolbooks than the ordered ones. Nearly the half of those who ordered the two schoolbooks in the greatest number of copies - Apáczai and Mozaik - uses other schoolbooks as a supplement. Almost half of the responders, 44 per cent uses teacher's book belonging to other schoolbook families. The sixth part of them also uses teacher's book belonging to other schoolbook. 29-29 per cent of those who ordered the schoolbook of Apáczai and Mozaik use teacher's books belonging to other schoolbook families. This series of data show several problems of wasting, conceptional contradiction in
teaching, the lack of harmony between the parties placing the order and the users etc. can be raised. This series of data itself is worth further investigations.

Index of satisfaction with the schoolbook can be that for how many years are the selected schoolbooks in use. According to the questionnaire the two-third of the teachers teaches in every class of the junior grades. A feedback on the congruence can be the selection of the same schoolbook family for the following cycle. Out of the three most frequently used schoolbooks in the 2009-10 school year the Apáczai was re-selected in 66 per cent, the Mozaik in 15 per cent and the Műszaki in 68 per cent after four years of use. However, one must be careful with this data, as the selection of the schoolbook can also be motivated by the professional interest beside the satisfaction. Another component causing further difficulties is that the trial of two-three schoolbook families would demand eight-twelve years, and when the schoolbook is marketed is still a question. All this requires differentiated problem management and ample circumspection.

Only fifth of the responders selected the schoolbooks upon professional considerations. Data shows that the professional aspects should be strengthened in the selection, and first of all in the publication of the schoolbooks (for example against the market aspects). This is an extremely complex professional task, which equally requires the research of schoolbooks, high quality schoolbook stylistic revision and critics, knowledge of learning psychology, learning methodological 'awareness' activities, the improvement of the demand for the mother tongue culture and the appropriate professional critics of the liberated (or libertine?) schoolbook market.

## Further research tasks

I have gained data in my research by analysing large number of schoolbooks in large perspective. Due to the very large number of data their processing and interpretation is far from being complete, secondary analysis is required and is potential. Furthermore it would also be required to carry out detailed analyses at other fields than arithmetic. The analysis of some traditionally problematic fields of geometry also demands attention: like the changes of teaching of measuring, measurements, and/or the methods to form the notions of perimeter, area and volume. Although the present economic and situation of education politics are highly unfavourable for the publication of new mathematics schoolbooks, however the idea of such schoolbook families can be outlined on the basis of the experiences of this research, which can exploit the favourable experiences of the predecessors.

But the really important thing is that are the lessons that can be learnt from all these, and how they can be delivered to the legislation, to the authors of schoolbook, to the publishers, to the evaluators and remarkably to the users: to the teachers and students. I regard the study of schoolbook usage of the pedagogues and students highly important, which could provide useful information for those who deal with the schoolbooks, to the authors and publishers of the textbooks. Similarly I think it is necessary to study the competency, which could provide feedback for the evaluation of the schoolbooks. However, such research can only be imagined by large teams, as it is not only representative on the one hand, but it would also require large sample analyses, while on the other verification of the relations and correlations of the schoolbooks and the competency (productivity) can cause significant professional difficulties.

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[^0]:    ${ }^{1}$ In the common and decimal fraction chapters of every schoolbook only positive fractions are dealt with.

