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Laser Physics, Nonlinear Optics, and Spectroscopy Program

Development of Extremely Strong Longitudinal Electric Field by Focusing of Radially Polarized THz Pulses

PhD Thesis

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1. Introduction

It is well known that a strong longitudinal electric field and a small spot size are observed when radially polarized beams are tightly focused using a high numerical aperture parabolic mirror [1, 2], which opens up possibilities for application in particle acceleration [3-5]. The parabolic mirror, serving as a single optical element, effectively covers nearly the complete 4π solid angles for focusing of radially polarized beam into a significantly tighter focal spot [6-8]. Parabolic mirrors with high reflectivity are excellent candidates for focusing electromagnetic beams and pulses in the THz frequency range due to the absence of chromatic aberrations and the possibility of using them in a large numerical aperture configuration [9, 10]. They are commonly used in both on- and off-axis configurations.

In order to accurately determine the electric and magnetic field distributions for all polarization components under the condition of tight focusing, it is necessary to employ the vector diffraction theory. The Stratton−Chu vector diffraction method [11] provides more accurate results than the Richards-Wolf method [12], particularly in situations involving long wavelengths [13]. Focusing of linearly polarized, monochromatic electromagnetic plane waves [14, 15], and pulses [9] by a paraboloid is already elaborated based on the Stratton−Chu diffraction theory. A more comprehensive investigation recently reported into applying the vector field focusing properties of electromagnetic fields by a parabolic mirror, based on the Stratton−Chu integral formalism [16-18]. The existing literature does not provide a detailed theoretical and analytical study of the vector field focusing properties of radially polarized beams by on- and off-axis parabolic mirrors based on the Stratton−Chu integral representation. This thesis presents a rigorous derivation of the electric field obtained when a parabolic mirror focuses a radially polarized, monochromatic, flat-top beam based on the Stratton−Chu integrals. It was shown that a strong longitudinal electric field can be obtained at the focus when focusing geometry with a large numerical aperture is used. It is evident that the largest contribution to the longitudinal field at the focus originates from the waves arriving from the directions perpendicular to the symmetry axis, z, while waves coming from the vicinity of the vertex, V, practically have no contribution [19].

The focusing properties of radially polarized Gaussian beam with a high numerical aperture optical element has recently attracted many researchers because of a small spot size and a strong axial electric field components in focal region [20, 21]. Linearly polarized laser beams typically exhibit behavior consistent with the widely recognized Gaussian beam formula, which is derived from the paraxial wave equation governing the electric field [22, 23]. However, it is important to note that this is not the only solution attainable from Maxwell's equations through reasonable approximations. Kirk T. McDonalds has successfully deduced the formulae for a radially polarized Gaussian Vector Beam, also referred to as an Axicon Beam [24]. His approach relied upon the application of a paraxial approximation within the wave equation concerning the vector potential. This thesis introduces derived formulae concerning the radial and axial electric fields, as well as the azimuthal magnetic field of a radially polarized Gaussian vector beam focused by a perfectly reflecting on-axis parabolic mirror with an arbitrary numerical aperture based on the Stratton−Chu vector diffraction theory. The field properties at the focal point are analyzed, particularly for a case with a large numerical aperture. It verifies the importance of the longitudinal electric field component at high numerical aperture for achieving the intense field strengths needed for particle acceleration. In case of radially polarized vector Gaussian beam, it examined the effects of beam divergence on axial and radial distributions of the longitudinal electric field component, showing that divergent beams shift the physical focus and influence the field enhancement factor. It was also observed that the shift of physical focus relative to the geometrical focus along the longitudinal direction exhibits a linear dependence on the divergence [25].

THz pulses possessing an extremely high field strength are well-suited for the acceleration of charged particles due to their appropriate wavelength and temporal period. Numerous approaches have been proposed and demonstrated in the literature for the acceleration of particles using laser-driven techniques [26]. Furthermore, there have been reports of the direct acceleration of a freely moving electron in an infinite vacuum along the axis by means of a tightly focused, ultra-intense radially polarized laser beam [27, 28]. After, the development of efficient THz generation techniques [29] has made it possible to consider THz pulse-driven particle acceleration as a realistic option, which has encouraged research efforts to explore the use of THz radiation in enhancing electron energy and beam characteristics in high-brightness electron sources [30] and proton post-accelerators [31]. Currently, the utilization of THz pulse-driven particle acceleration in both vacuum [32] and waveguide structures [33] has emerged as a highly active and promising area of study.

The focused THz pulses could be a potential solution for efficient particle acceleration [34]. Single-cycle THz pulses are promising because of their advantageous wavelength and the available peak electric field at the MV⁄cm level [35, 36]. It is important to carefully consider the shape of the electric field of the low-frequency THz pulses at the focus for the case of particle acceleration [37]. The choice of the paraboloid mirror shape, whether annular or of arbitrary opening, can potentially impact the sharpness and dimensions of the focal spot. Consequently, a fully illuminated paraboloid having one contour is not the optimal focusing element if the goal is to achieve the maximal field component in the z direction for a fixed amount of input energy/beam power. Instead, a ring-like paraboloid segment to convert optimally the input radial THz field into axial component is proposed in this thesis. Thus, this thesis proposes a new setup and method for tightly focusing radially polarized THz pulses to generate a strong accelerating electric field suitable for vacuum electron acceleration. The designed setup consists of a reflaxicon for beam shaping to minimize power loss, a ring-like segment of an on-axis parabolic mirror for tightly focusing the radially polarized pulses, a Segmented Half-Waveplate (SHWP) for converting linear pump polarization to radial, and a Segmented Nonlinear Material (SNM) for generating the radially polarized THz beam. The electric field distribution in the focal region is determined using the Stratton−Chu vector diffraction theory. The field enhancement factor, which is the ratio of the longitudinal electric field strength to the input radial electric field component, significantly exceeds 1000. By focusing radially polarized 1 ps single-cycle THz pulses with 3 mJ pulse energy, calculations predicted accelerating electric field component in the order of 100 MV/cm, which is well suitable for particle acceleration. An energy gain on the order of MeV by an electron moving at nearly the speed of light was obtained from the study of acceleration using a simple model. The thesis contributes significantly to particle acceleration by providing theoretical foundations and practical methodologies for generating strong accelerating fields using focused radially polarized electromagnetic pulses [38].

The thesis suggests several future research directions. This includes further optimizing the focusing setup and exploring different geometries to enhance field intensities. The thesis also recommends expanding and verifying the theoretical models used in this study through experimental setups to establish a solid foundation for future advancements in particle acceleration. Additionally, the detailed study of the acceleration mechanism will be a future task, which involves exploring the interaction between the pulses and particles in more depth, improving the design of the focusing setup, and potentially increasing the efficiency and effectiveness of the particle acceleration process.

2. Scientific Goals

The main goal of the thesis was to investigate theoretically the generation of extremely strong longitudinal electric field by focusing radially polarized monochromatic laser beams and THz pulses using a paraboloid mirror or its segment with perfect reflectance. This involved analyzing the axial and radial distributions of the longitudinal electric field and the pulse characteristics in the focal region, aiming to attain high field intensities suitable for particle acceleration applications.

The specific objectives of the thesis were:

- To derive general formulae for the electric field produced by focusing of radially polarized, monochromatic, flat-top beam using a parabolic mirror. This involves a rigorous theoretical analysis using the Stratton−Chu vector diffraction theory to examine the distribution of the electric field, especially the characteristics of the longitudinal and radial electric field components for particle acceleration applications.
- To develop general formulae for the electric and magnetic field characteristics of a focused radially polarized Gaussian vector beam by a parabolic mirror, based on the Stratton−Chu vector diffraction method. This objective focuses on different numerical apertures and incident beam divergences to determine axial and radial distributions of the longitudinal electric field component, essential for particle acceleration.
- To validate and compare the derivation of the electric field distribution formulae, which are based on the Stratton−Chu vector diffraction theory, with a theory based

on Rayleigh–Sommerfeld and Richards−Wolf models in the focal region, specifically for a case involving large NA and short wavelength regimes.

- To generate extremely strong accelerating electric fields by focusing of radially polarized THz pulses with a parabolic mirror ring. The objective here is to create a novel setup using a reflaxicon and a ring-like segment of a parabolic mirror to achieve high field intensities suitable for vacuum electron acceleration. The calculation of electric field distribution in the focal region was performed by the Stratton–Chu vector diffraction method.
- To determine an energy gain on the order of MeV by an electron moving at nearly the speed of light from the study of the acceleration using a simple model and expecting the energy gain to depend on the relative motion direction (same and opposite) of the electron and the sweeping velocity of the field maxima.

3. New Scientific Results

1. Using the Stratton-Chu vector diffraction method, I derived formulae regarding the spatial distribution of the electric and magnetic field strength of radially polarized monochromatic laser beams focused with a parabolic mirror. The derivations were performed for both a Flat-top input beam and a vector beam with a Gaussian intensity distribution. I derived relationships regarding the spatial distribution of the electric field components of a radially polarized beam focused by a system consisting of axicons and a parabolic mirror ring designed for accelerating charged particles. The transverse intensity distribution of the beam arriving at the parabolic mirror ring was considered in three different cases that were relevant from a practical point of view. All of my above-mentioned derivations are universal: they can be applied to any wavelength/focal length ratio and arbitrary NA. [S1,S2,S3]
2. In the cases mentioned in the first thesis point, I analyzed and interpreted in detail the spatial distribution of electric and magnetic field strength components in the vicinity of the focus – with particle acceleration applications in mind – with special attention on the longitudinal component of electric field strength. In each case, I determined the scale law for the longitudinal electric field strength in focus, the variables of which are the parameters of the input beam, the focal length and the characteristic parameter of the geometry of focusing. I looked for the optimal geometry for achieving maximum longitudinal field strength in each case. I showed that in the case of practical, optimized focusing, the ratio of the longitudinal electric field strength in focus to the input radial electric field component exceeds 1000. I showed that by optimized focusing of today's THz pulses with energy of the order of mJ, longitudinal electric field strength of the

order of 100 MV/cm can be achieved, which is a remarkable fact from the point of view of particle acceleration applications. [S1,S2,S3]

3. The longitudinal and transverse distribution of the longitudinal component of the electric field of a radially polarized, monochromatic vector beam with a Gaussian intensity distribution focused with a parabolic mirror was investigated in the vicinity of the focus as a function of the divergence of the input beam. I showed that the real focus (the maximum location of the longitudinal field strength component) is shifted relative to the geometric focus along the axis of symmetry of the beam. I showed that this shift is a linear function of the divergence of the input beam. I found that the maximum longitudinal electric field component value achievable by focusing is a monotonically decreasing function of the input beam divergence, and that this dependence is significant. Furthermore, I showed that in the transverse distribution of the longitudinal component of the electric field amplitude, the ratio of the sub maxima to the principal maximum value increases with the degree of beam divergence. [S2]
4. I applied a numerical code to simulate the post-acceleration of relativistic electrons with the longitudinal electric field of THz pulses focused with a parabolic mirror ring. According to the simulations, assuming a THz pulse energy of 3 mJ, a pulse length of 1 ps, and a parabolic mirror ring with a focal length of 5 cm illuminated at 20° and 40° viewing angles, electrons gain of 0.28 MeV/0.23 MeV (20°) and 0.63 MeV/0.36 MeV (40°) are available depending on whether the direction of motion of electrons moving at near the speed of light is the same/opposite to the direction of the electric field's sweeping velocity. [S3]

List of Publications

Publications in the topic of thesis

[S1] **Z. T. Godana**, J. Hebling, and L. Pálfalvi, "Focusing of Radially Polarized Electromagnetic Waves by a Parabolic Mirror," in Photonics, vol. 10, no. 7: MDPI, p. 848, 2023.

[S2] L. Pálfalvi, **Z. T. Godana**, and J. Hebling, "Electromagnetic Field Distribution and Divergence-Dependence of a Radially Polarized Gaussian Vector Beam Focused by a Parabolic Mirror," *arXiv preprint arXiv:2406.00795,* 2024.

[S3] L. Pálfalvi, **Z. T. Godana**, G. Tóth, and J. Hebling, "Generation of extremely strong accelerating electric field by focusing radially polarized THz pulses with a paraboloid ring," Optics & Laser Technology, vol. 180, p. 111554, 2025.

Conference contributions

[E1] **Z. T. Godana**, J. Hebling, and L. Pálfalvi, " Radially Polarized Electromagnetic Wave Focused by a Segment of a Parabolic Mirror," in Proceedings of the 6th International Conference on Optics, Photonics and Lasers (OPAL), 2023, pp64. (ISBN: 978-84-09-48335-8).

[E2] L. Pálfalvi, **Z. T. Godana**, G. Tóth, and J. Hebling, " Extremely High Electric Field for Electron Acceleration by Focusing of THz Pulses with Paraboloid Mirrors," in 10th Optical Terahertz Science and Technology (OTST), 2024, Paper WeA5.106.

[E3] L. Pálfalvi, **Z. T. Godana**, G. Tóth, and J. Hebling, " Production of Extremely Strong Accelerating Electric Field by Focusing THz Pulses with a Paraboloid Ring," in Conference on Lasers and Electro-Optics (CLEO), 2024: Optica Publishing Group, Paper JTu2A.63. (ISBN: 978-1-957171-39-5).

[E4] **Zerihun Tadele Godana**, László Pálfalvi, György Tóth and János Hebling, " Extremely Strong Longitudinal Electric Field by Focusing of Radially Polarized THz Pulses with a Paraboloid Ring," in Proceedings of the Doctoral Students' Conference for the Discussion of Optical Concepts (DoKDoK), 2024, Friedrich Schiller University Jena, Abbe School of Photonics, Germany, pp72.

References

[1] R. Dorn, S. Quabis, and G. Leuchs, "Sharper focus for a radially polarized light beam," *Physical review letters,* vol. 91, no. 23, p. 233901, 2003.

[2] H. Dehez, A. April, and M. Piché, "Needles of longitudinally polarized light: guidelines for minimum spot size and tunable axial extent," *Optics Express,* vol. 20, no. 14, pp. 14891-14905, 2012.

[3] V. Marceau, C. Varin, and M. Piché, "Validity of the paraxial approximation for electron acceleration with radially polarized laser beams," *Optics letters,* vol. 38, no. 6, pp. 821-823, 2013.

[4] S. Payeur *et al.*, "Generation of a beam of fast electrons by tightly focusing a radially polarized ultrashort laser pulse," *Applied Physics Letters,* vol. 101, no. 4, 2012.

[5] L. J. Wong and F. X. Kärtner, "Direct acceleration of an electron in infinite vacuum by a pulsed radially-polarized laser beam," *Optics express,* vol. 18, no. 24, pp. 25035-25051, 2010.

[6] N. Davidson and N. Bokor, "High-numerical-aperture focusing of radially polarized doughnut beams with a parabolic mirror and a flat diffractive lens," *Optics Letters,* vol. 29, no. 12, pp. 1318-1320, 2004.

[7] A. April and M. Piché, "4π Focusing of TM 01 beams under nonparaxial conditions," *Optics express,* vol. 18, no. 21, pp. 22128-22140, 2010.

[8] J. Stadler, C. Stanciu, C. Stupperich, and A. Meixner, "Tighter focusing with a parabolic mirror," *Optics letters,* vol. 33, no. 7, pp. 681-683, 2008.

[9] H. Zhang *et al.*, "Parabolic Mirror Focusing of Spatiotemporally Coupled Ultrashort Terahertz Pulses," *IEEE Photonics Journal,* vol. 15, no. 4, pp. 1-8, 2023.

[10] M. Wichmann *et al.*, "Terahertz plastic compound lenses," *Appl. Opt.,* vol. 52, no. 18, pp. 4186-4191, 2013/06/20 2013, doi: 10.1364/AO.52.004186.

[11] J. A. Stratton and L. Chu, "Diffraction theory of electromagnetic waves," *Physical review,* vol. 56, no. 1, p. 99, 1939.

[12] B. Richards and E. Wolf, "Electromagnetic diffraction in optical systems, II. Structure of the image field in an aplanatic system," *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences,* vol. 253, no. 1274, pp. 358-379, 1959.

[13] X. Zeng and X. Chen, "Characterization of tightly focused vector fields formed by off-axis parabolic mirror," *Optics Express,* vol. 27, no. 2, pp. 1179-1198, 2019.

[14] P. Varga and P. Török, "Focusing of electromagnetic waves by paraboloid mirrors. I. Theory," *JOSA A,* vol. 17, no. 11, pp. 2081-2089, 2000.

[15] P. Varga and P. Török, "Focusing of electromagnetic waves by paraboloid mirrors. II. Numerical results," *JOSA A,* vol. 17, no. 11, pp. 2090-2095, 2000.

[16] T. Phan *et al.*, "High-efficiency, large-area, topology-optimized metasurfaces," *Light: Science & Applications,* vol. 8, no. 1, p. 48, 2019.

[17] J. Grand and E. C. Le Ru, "Practical implementation of accurate finite-element calculations for electromagnetic scattering by nanoparticles," *Plasmonics,* vol. 15, no. 1, pp. 109-121, 2020.

[18] H. Guerboukha *et al.*, "Efficient leaky-wave antennas at terahertz frequencies generating highly directional beams," *Applied Physics Letters,* vol. 117, no. 26, 2020.

[19] Z. T. Godana, J. Hebling, and L. Pálfalvi, "Focusing of Radially Polarized Electromagnetic Waves by a Parabolic Mirror," in *Photonics*, 2023, vol. 10, no. 7: MDPI, p. 848.

[20] G. Endale, D. Mohan, and S. Yadav, "Focusing of radially polarized bessel gaussian and hollow gaussian beam of high NA to achieve super resolution," *Optik,* vol. 253, p. 168586, 2022.

[21] S. W. Jolly, "Modeling the focusing of a radially polarized laser beam with an initially flat-top intensity profile," *Journal of the Optical Society of America A,* vol. 41, no. 7, pp. 1390-1396, 2024.

[22] B. E. Saleh and M. C. Teich, *Fundamentals of photonics*. john Wiley & sons, 2019.

[23] B. D. Guenther, *Modern optics*. OUP Oxford, 2015.

[24] K. T. McDonald, "Axicon Gaussian laser beams," *arXiv preprint physics/0003056,* 2000.

[25] L. Pálfalvi, Z. T. Godana, and J. Hebling, "Electromagnetic Field Distribution and Divergence-Dependence of a Radially Polarized Gaussian Vector Beam Focused by a Parabolic Mirror," *arXiv preprint arXiv:2406.00795,* 2024.

[26] L. J. Wong *et al.*, "Laser-induced linear-field particle acceleration in free space," *Scientific reports,* vol. 7, no. 1, p. 11159, 2017.

[27] C. Varin *et al.*, "Direct electron acceleration with radially polarized laser beams," *Applied Sciences,* vol. 3, no. 1, pp. 70-93, 2013.

[28] S. W. Jolly, "On the importance of frequency-dependent beam parameters for vacuum acceleration with few-cycle radially polarized laser beams," *Optics Letters,* vol. 45, no. 14, pp. 3865-3868, 2020.

[29] J. A. Fülöp, L. Pálfalvi, M. C. Hoffmann, and J. Hebling, "Towards generation of mJ-level ultrashort THz pulses by optical rectification," *Optics express,* vol. 19, no. 16, pp. 15090-15097, 2011.

[30] J. Hebling, J. Fülöp, M. Mechler, L. Pálfalvi, C. Tőke, and G. Almási, "Optical manipulation of relativistic electron beams using THz pulses," *arXiv preprint arXiv:1109.6852,* 2011.

[31] L. Pálfalvi, J. A. Fülöp, G. Tóth, and J. Hebling, "Evanescent-wave proton postaccelerator driven by intense THz pulse," *Physical Review Special Topics-Accelerators and Beams,* vol. 17, no. 3, p. 031301, 2014.

[32] Z. Tibai, S. Turnár, G. Tóth, J. Hebling, and S. W. Jolly, "Spatiotemporal modeling of direct acceleration with high-field terahertz pulses," *Optics express,* vol. 30, no. 18, pp. 32861-32870, 2022.

[33] S. Turnár, G. Krizsán, J. Hebling, and Z. Tibai, "Waveguide structure based electron acceleration using terahertz pulses," *Optics Express,* vol. 30, no. 15, pp. 27602-27608, 2022.

[34] Z. Tibai *et al.*, "Relativistic electron acceleration by focused THz pulses," *Journal of Physics B: Atomic, Molecular and Optical Physics,* vol. 51, no. 13, p. 134004, 2018.

[35] B. Zhang *et al.*, "1.4-mJ high energy terahertz radiation from lithium niobates," *Laser & Photonics Reviews,* vol. 15, no. 3, p. 2000295, 2021.

[36] G. Tóth, G. Illés, G. Nazymbekov, N. Mbithi, G. Almási, and J. Hebling, "Possibility of CO2 laser-pumped multi-millijoule-level ultrafast pulse terahertz sources," *Scientific Reports,* vol. 14, no. 1, p. 999, 2024.

[37] S. Turnár, J. Hebling, J. Fülöp, G. Tóth, G. Almási, and Z. Tibai, "Design of a THz-driven compact relativistic electron source," *Applied Physics B,* vol. 127, pp. 1-7, 2021.

[38] L. Pálfalvi, Z. T. Godana, G. Tóth, and J. Hebling, "Generation of extremely strong accelerating electric field by focusing radially polarized THz pulses with a paraboloid ring," *Optics & Laser Technology,* vol. 180, p. 111554, 2025.