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ASPECTS OF

PARAMETRIC ARCHITECTURAL DESIGN

IN THE CONTEXT OF

GRAPH THEORY

AND

COORDINATE SYSTEM TRANSFORMATIONS

PHD THESIS

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1 INTRODUCTION

In this thesis work I examined parametric architecture and its applications on numerous ways. During this research I got an overall view about of this branch of architecture and I used this knowledge not only in further research but in design tasks and in education too. Because of this my research is widespread in the field of parametric architectural design.

I researched and classified the parametric design tools and I also assigned its connections to traditional architectural design. I created a plug-in for Grasshopper, the parametric software I primarily use, which contains coordinate system transformation tools partly based on another similar tool, Formex Algebra. I also added more tools based on the specifications of parametric design. With these tools I created numerous domes and tested their statical behavior, and analyzed it compared to their graph characteristics.

I teach parametric design for architecture students for five years. During this time, I researched and experimented how to teach this subject on an interesting and comprehensive way.

I also included in this thesis work some designs I made with parametric tools, which are connected to my research. These works represent the advantages and applications of parametric design well.

1.1 PROBLEM STATEMENT, PURPOSE AND SCOPE

The main focus of this research is parametric architecture. The aim is to understand the cognitive model, the tools and the applications of parametric design, improve the theoretical concepts and add to the practical utilization.

Parametric design is a new, specific field of architecture. It is an interdisciplinary field, geometry, mathematics and programming are also fields of science which are necessary for parametric design. In this research all of these play a part.

The first intent of this research was to understand parametric design better, and design a clear classification system for parametric design techniques which helps this understanding.

The next step was to utilize the possibilities of parametric design and to develop a dome and vault creating tool for parametric design, in a user friendly environment, with a calculation method which makes easy application possible.

The next goal was to create new, similar tools for other curved surfaces.

The next aim was to determine if the “pattern” of a structure can affect its statical behavior on a way which can be clearly determined by its graph characteristics.

The final goal of this research was to create a syllabus for architectural students with which they are able to acquire parametric design easily and effectively, and makes possible to implement parametric design to the curriculum of architectural students.

2 THESIS

2.1 THESIS 1 – CLASSIFICATION SYSTEM OF PARAMETRIC PATTERNS

I have created a theoretical based classification system for parametric patterns and structures.

I have studied the existing groupings and classifications and created a clear, more comprehensive system. The former classifications were usually based on the execution of parametric design techniques. I have created a theoretical based classification system which gives a clear overview of parametric design techniques. This classification system can help architects and designers to understand and apply parametric design in a more efficient and concise way.

2.1.1 SUBTHESIS 1 ABOUT PREVIOUS CLASSIFICATIONS

I have researched and compared the existing classification systems and identified their imperfections.

The problems were primarily based on the fact, that these works mostly concentrated on the practical solutions. Because of this they were not totally consequent and comprehensive.

2.1.2 SUBTHESIS 2 ABOUT PARAMETRIC WORKFLOW

I have created a model to describe the workflow of parametric design.

It was necessary to be able to create an adequate classification system. Thanks to this the classification system matches the parametric workflow.

2.1.3 SUBTHESIS 3 ABOUT GRAPH CHARACTERISTICS AND REGULAR AND IRREGULAR PATTERNS

I have classified the patterns based on their graph representation. I have created an exact solution to differentiate regular and irregular patterns based on their graph characteristics, and I have extended this solution to spatial structures too.

2.1.4 PUBLICATIONS

PAPER IN CONFERENCE PROCEEDINGS (IN HUNGARIAN)

Classification of Parametric Design Techniques Based on Architectural Point of View, in Hungarian, Réka Sárközi, IV. Conference Proceedings Interdiszciplináris Doktorandusz Konferencia 2015, pp. 449-464. ISBN:978-963-642-830-3

Paper in conference proceedings

Classification of Parametric Design Techniques, Réka Sárközi, Péter Iványi, Attila Béla Széll, Proceedings of the Fifteenth International Conference on Civil, Structural and Environmental Engineering Computing, 2015, Paper 227, pp. 1-15. ISBN:978-1-905088-63-8

Conference abstract

Classification of Parametric Design Techniques, Réka Sárközi, Péter Iványi, Attila Béla Széll, 11th Miklós Iványi International PhD&DLA Symposium, 2015, Conference abstract, pp. 106., ISBN:978-963-642-876-1

Paper in conference proceedings

Classification of Parametric Design Techniques, Types of Surface Patterns, Réka Sárközi, Péter Iványi, Attila Béla Széll, CAADence 2016 International Workshop and Conference, Conference proceedings, pp. 253-258., DOI: 10.3311/CAADence.1661

Paper in scientific journal

Classification of Parametric Design Techniques, Types of Surface Patterns, Réka Sárközi, Péter Iványi, Attila Béla Széll, Pollack Periodica: An International Journal for Engineering and Information Sciences 12(2) pp. 173-180. (2017)

Paper in conference proceedings

Historical Preludes of Parametric Design Techniques, Réka Sárközi, Péter Iványi, Attila Béla Széll, Places and Technologies 2019: The 6. th International Academic Conference on Places and Technologies, pp. 524-528.

2.2 THESIS 2 – FORMEX ALGEBRA ADAPTATION

I have proved it is possible to implement Formex algebra into modern software. I have created a tool in Grasshopper based on the mathematical solutions used in Formex algebra. I have created a new calculation method which can transform grid structures given in Descartes Coordinate System into domes, vaults or polar structures using ellipsoidal, cylindrical or polar coordinates.

Formex algebra is a very efficient tool to create dome and vault grid structures because it uses coordinate system transformation, at the same time it requires programming skills which is not very common among architects. In modern parametric software it can reach a wider user community.

I have implemented the coordinate system transformation method which can create dome structures from triangular grids. I have created a solution which is more user friendly than the original one. The user can define the properties of the structure with input parameters. They do not have to take a lot of mathematical calculations to reach the desired outcome like in Formex algebra.

2.2.1 SUBTHESIS 1 ABOUT ELLIPSOIDAL COORDINATES

I have improved the Formex algebra formulation by using ellipsoidal coordinates not spherical coordinates to create dome structure.

It makes possible to create domes with different height and radius. This solution allows for a more versatile shape definition.

2.2.2 SUBTHESIS 2 ABOUT MODIFIED STRUCTURES

I have created various modified grid structures based on the ellipsoidal dome structure. In these modified structures one or two angular coordinates are replaced with planar coordinates.

Because of these structures are created based on ellipsoidal coordinates, these modified structures fit into the same system, their not modified edges can be connected with ellipsoidal structures, their modified edges can be connected with other modified structures. The vault and polar grid tool also contains this kind of modification.

2.2.3 SUBTHESIS 3 ABOUT THE CUPOLA TOOL

I have created a new Formex algebra form that was not covered by the original approach. This is one of the modified dome structures, I named it 'Cupola' tool.

2.2.4 SUBTHESIS 4 ABOUT TRIANGULAR GRIDS

I have implemented the triangular grid based solution into the polar grid and the vault tool, which was not covered by the original Formex Algebra.

For this I have created a solution, which calculates the skew coordinates of a triangular grid structure. With a coordinate system transformation, the triangular grid is transformed to fit into a rectangle. This grid can be transformed with the same calculations to dome vault or polar grid structures just like a rectangular grid. With this solution I have created a universal solution to transform triangular grids just like rectangular grids.

2.2.5 PUBLICATIONS

CONFERENCE ABSTRACT

Formex Algebra Adaptation into Parametric Design Tools, Réka Sárközi, Péter Iványi, Attila Béla Széll, 13th Miklós Iványi International PhD&DLA Symposium, 2017, Conference abstract, pp. 114., ISBN:978-963-642-780-1

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Formex Algebra Adaptation into Parametric Design Tools: Dome Structures, Réka Sárközi, Péter Iványi, Attila Béla Széll, Conference Proceedings, Innsbruck Austria Jan 25-26, 2018, 20 (1) Part XV, pp. 1693-1697.

PAPER IN SCIENTIFIC JOURNAL

Formex Algebra Adaptation into Parametric Design Tools: Dome Structures, Réka Sárközi, Péter Iványi, Attila Béla Széll, World Academy of Science, Engineering and Technology International Journal of Architectural and Environmental Engineering Vol:12, No:1, 2018, pp. 62-66 DAI: urn: dai:10.1999/1307-6892/10008543

2.3 THESIS 3 – ROTATIONAL GRID TOOL

I have created a new calculation method in Grasshopper which can transform any grids given in Descartes coordinate system to rotational grids. The rotational surface can be given with its generating curve.

2.3.1 SUBTHESIS 1 ABOUT DIFFERENT MATHEMATICAL CALCULATIONS

I have created two tools with different mathematical calculations. The calculation of the height and thickness of these structures are different.

The first one deeply lies on the basis of the cylindrical coordinates, the second one follows the properties of the generating curve more. The two tools calculate the height and the thickness of the structures differently, they can be used for different purposes.

2.3.2 SUBTHESIS 2 ABOUT TRIANGULAR GRID VERSION

I have created a version of both tools which transforms triangular grids to rotational grids.

2.3.3 PUBLICATIONS

CONFERENCE ABSTRACT

Parametric vault design tools based on Formex algebra, Réka Sárközi, Péter Iványi, Attila Béla Széll, Tenth International Conference on Engineering Computational Technology, 2018, Conference abstract

PAPER ACCEPTED BY SCIENTIFIC JOURNAL

Formex Algebra Adaptation into Parametric Design Tools and Rotational Grids, Réka Sárközi, Péter Iványi, Attila Béla Széll, Pollack Periodica: An International Journal for Engineering and Information Sciences

2.4 THESIS 4 – GRAPH CHARACTERISTICS OF DOME STRUCTURES

I have proved that the statical behavior of single layer truss-grid domes can be predicted by the calculation of specific graph characteristics of the dome.

I have proved that the application of fix joints, which can transfer both axial and shear forces, torsional and bending moments changes this behavior and the statical behavior of these dome structures is not clearly in correlation with their studied graph characteristics.

I have proved that studied graph characteristics cannot be used in the same form to predict the statical behavior of double layer truss-grid domes because the studied graph characteristics cannot represent the properties of spatial structures clearly.

2.4.1 PUBLICATIONS

PAPER IN CONFERENCE PROCEEDINGS

Correlation Between the Statical Behaviour of Dome Structures and their Graph Characteristics, Réka Sárközi, Péter Iványi, Attila Béla Széll, Slovak University of Technology in Bratislava, Faculty of Mechanical Engineering, 18th Conference on Applied Mathematics APLIMAT 2019 Proceedings, pp. 1035-1040.

CONFERENCE ABSTRACT

Graph Characteristics of Dome Structures, Réka Sárközi, Péter Iványi, Attila Béla Széll, Conference Proceedings V. INTERNATIONAL ARCHITECTURAL DESIGN CONFERENCE ARCHDESIGN '19, 2019

2.5 THESIS 5 – TEACHING PARAMETRIC DESIGN

I have proved that the education of parametric design can be imported to the curriculum of architect students despite the fact that it requires special skills which are not common among architects.

I have identified the skills which have to be improved for architectural students to be able to acquire parametric design and I have built in exercises to the curriculum which improve these skills.

I have proved that it is necessary to teach the cognitive model – Parametric Design Thinking – and the actual application of parametric software together to earn success with teaching parametric design for architecture students. I have implemented this method into the curriculum.

2.5.1 PUBLICATIONS

CONFERENCE ABSTRACT

Case Study: Using Parametric Tools in Architectural Practice, Réka Sárközi, Péter Iványi, Attila Béla Széll, 12th Miklós Iványi International PhD&DLA Symposium, 2016, Conference abstract, pp. 105., ISBN:978-963-429-094-0

CONFERENCE ABSTRACT

Vasarely Inspired Pavilions - How Architecture Students Can Use Op-art, Attila Béla Széll, Réka Sárközi, 12th Miklós Iványi International PhD&DLA Symposium, 2016, Conference abstract, pp. 114., ISBN:978-963-429-094-0

PAPER IN CONFERENCE PROCEEDINGS

Methods of Teaching Parametric Design for Architectural Students, Réka Sárközi, Péter Iványi, Attila Béla Széll, Conference Proceedings V. INTERNATIONAL ARCHITECTURAL DESIGN CONFERENCE ARCHDESIGN '18, pp. 25-29. (2018)

LECTURE NOTES

Parametric Design: Graphical Algorithm Editing for Architects in Grasshopper® in Hungarian, Réka Sárközi, 2019, ISBN: 978-963-429-387-3

3 ABOUT PARAMETRIC DESIGN

Parametric design is a new and special type of architecture for various reasons. Nowadays it is not commonly used in architectural practice, yet the application of this field of architecture spreads quickly. Parametric design requires skills and capabilities, which are not commonly necessary for architects. To be able to use these new tools, architects also have to acquire parametric design thinking, which is the cognitive model of parametric design what is based on the topological built-up of the structures.

3.1 WHAT IS PARAMETRIC DESIGN?

Parametric design is a design method which uses computers and algorithms to create three dimensional models. The advantage of parametric design is that it is very useful for optimization and form finding, because it is possible to get different versions of a model with changing the input parameters of the algorithm. It is possible to handle a lot of different spatial elements together, and with its mathematical and geometrical commands it is possible to create shapes and forms which would be very hard to handle with traditional architectural tools.

Its disadvantage is that the understanding of algorithms is necessary to be able to use it efficiently. If it is used in cases which cannot take its advantages, it is a more circumstantial and difficult solution.

3.2 APPLICATIONS OF PARAMETRIC DESIGN

Parametric Design has a wide range of design tools, introduced in a lot of resources [1-6]. It is also applied in many fields of architecture [7] and other design fields, like object design [3]. With the usage of parametric design architects and designers are able to adapt the design to a lot of parameters, and the computing capacity of a computer is a very strong tool to make this possible.

Parametric Design has many applications. Besides the possibility of creating parametric patterns and shapes for architectural purposes [1, 2, 8], it can be used to optimize functional connections in architectural and urban design [9, 10], make energy simulations [11, 12], or statical optimization [13]. It also allows for designers the usage of a lot of new shapes and forms.

3.3 PARAMETRIC DESIGN THINKING

Architects, engineers and designers still keep changing the definition of “Parametric design thinking”. According to Oxman, this change is fundamentally based on the changing and evolving parametric design tools, and on the ways how designers use them. [14] This means that how the tools and abilities of designers change it changes their definition of Parametric design thinking.

Learning the application of parametric design tools without the theory of parametric design thinking is possible but pointless. The learning of the application of parametric design software and the theory of parametric design thinking have to be parallel, continuous and related.

Architects learn the way of design thinking on the university, and using it constantly in their practice. They learn how to use their creativity to solve architectural problems and create spaces and buildings. They learn the method of evolution, reflection and re-editing and they identify it as a creative and impressive process. They solve architectural tasks with the modification and adaptation of their typological knowledge [14]. By using parametric design and making the parametric scheme of a building architects have to design a rule structure [14], and for the first sight it seems to be the complete opposite of what they learned and made for years. Designers have to think in rules, formulas and schemes, and it looks like that this design process neglects creativity. You can see the scheme of conventional and parametric design process in Figure 3.1.

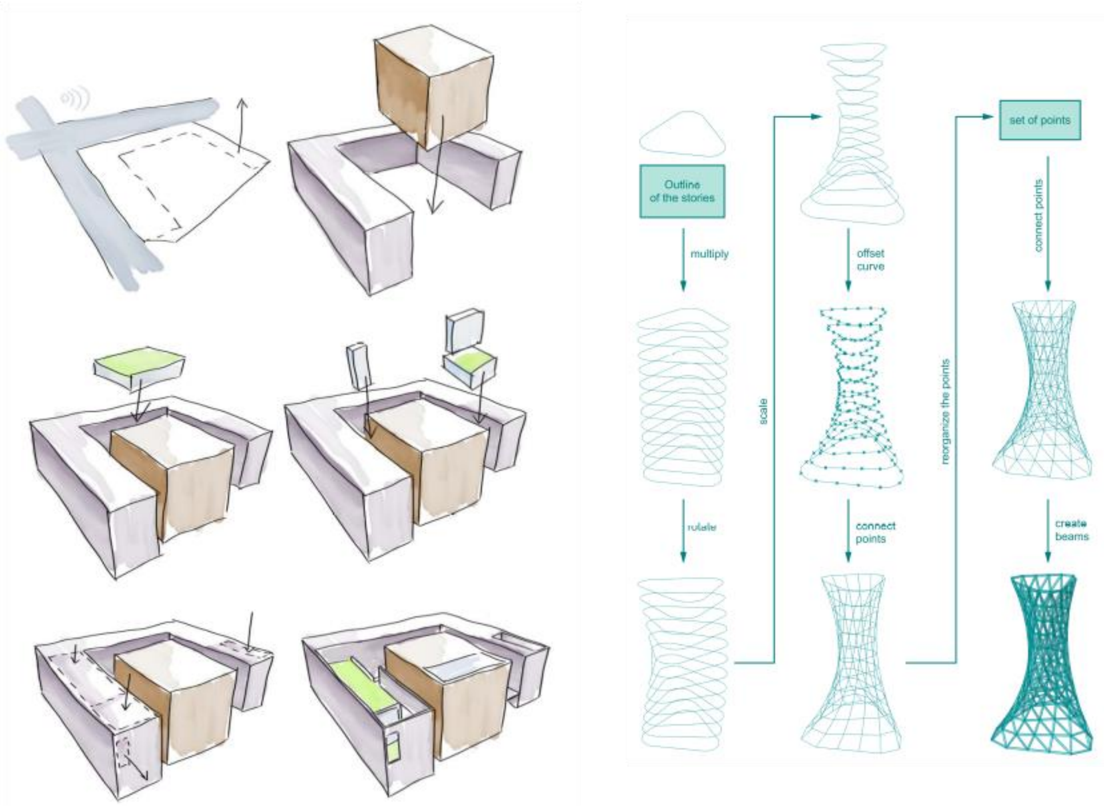


FIGURE 3.1 SCHEME OF TRADITIONAL AND PARAMETRIC ARCHITECTURAL DESIGN PROCESS

However, the creative process of parametric design is the creation of the rule system, the parametric scheme itself in order to create the wanted shapes. The first step of learning parametric design thinking is the understanding, that the design effects the rules and not the rules effect the design.

3.4 PARAMETRIC DESIGN TOOLS

To use parametric design, specific software is necessary. The examples in this thesis work were created with the 3D-modelling software Rhino 3D [15], and its plug-in Grasshopper [16]. There are a lot of other plug-ins available for Grasshopper, so it is possible to join it with the most popular architectural design software.

The different parametric design software can differ in lot of ways but they usually have a graphical algorithm editor, and the basic logic of them is the same. They use mathematical and geometrical tools for creation and transformation. It is possible to create freeform surfaces, irregular patterns, use computation-heavy solutions and specific algorithms.

3.4.1 RHINO

Rhino 3D is a NURBS (Non-uniform rational basis spline) based [17] 3D modelling software. Its advantage is that NURBS-surfaces do not approach curved surfaces with small planar surfaces, they are actually curved. This makes it very useful in the creation of organic models.

3.4.2 GRASSHOPPER

Grasshopper is a graphical algorithm editor which works together Rhino 3D. It uses the syntax of Rhino. With Grasshopper it is possible to give algorithm controlled commands to Rhino as it can be seen in Figure 3.2.

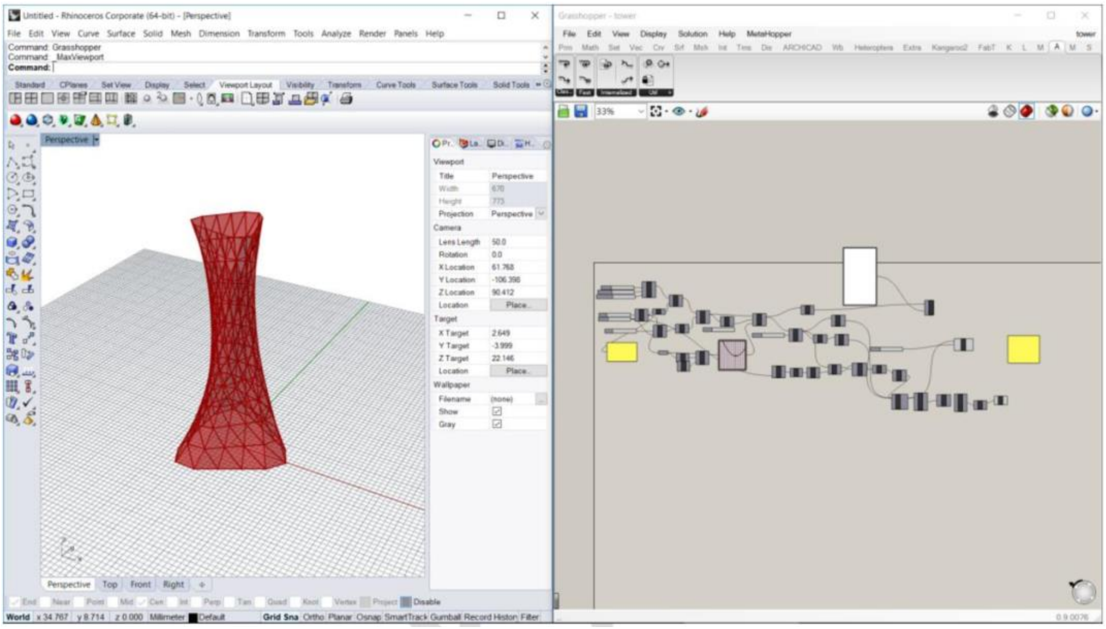


FIGURE 3.2 THE WINDOWS OF RHINO AND GRASSHOPPER

3.4.3 PYTHON IN RHINO AND GRASSHOPPER

Although Grasshopper is a very powerful tool in parametric design, it has its limitations. It requires additional plug-ins to be able to create loops, and even then the execution is very slow. Some mathematical calculations require a lot of computation capacity too. In these cases, it is possible to create scripts in Grasshopper. Grasshopper can use three programming languages, C#, Visual Basic and Python. Because of the more user friendly syntax in this thesis work Python is used. It is able to use the functions of Rhino with the library RhinoScriptSyntax [18] and the functions of Grasshopper with the library GhPython [19], as it can be seen in Figure 3.3.

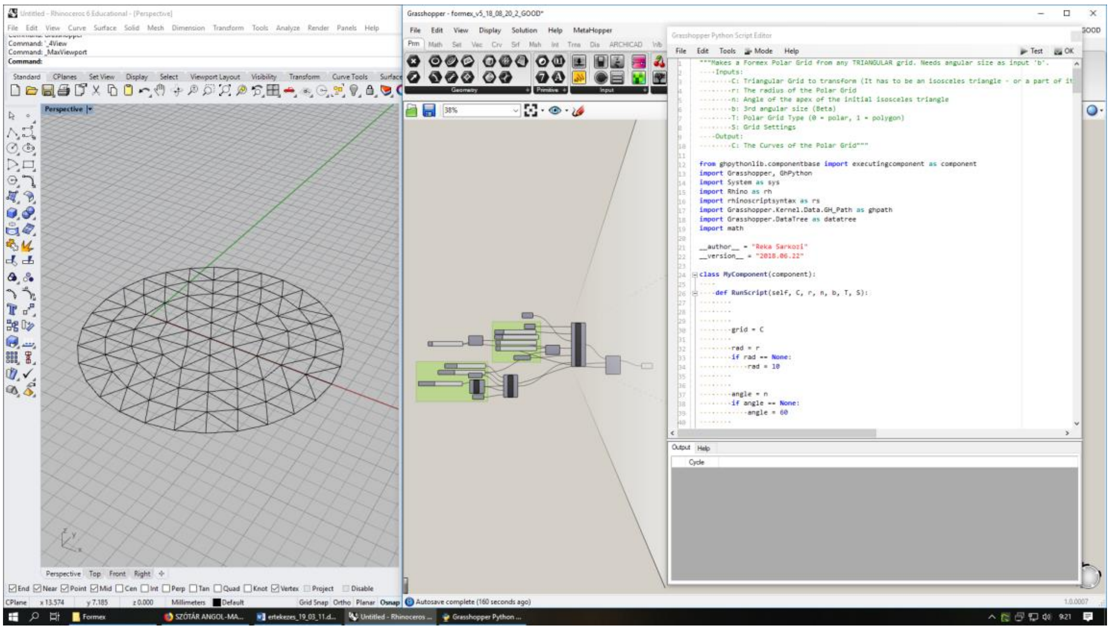


FIGURE 3.3. CREATING SCRIPTS IN GRASSHOPPER

3.5 HISTORICAL PRELUDES OF PARAMETRIC DESIGN TECHNIQUES

3.5.1 EVOLUTION OF PARAMETRIC DESIGN

In the previous chapter the differences between traditional and parametric architectural design were introduced. At the same time traditional architectural design process is mostly based on the typological knowledge of the architects. They investigate the former solution for a similar task, analyze these based on the needs and circumstances, use and combine the best solutions and modify them to fit to the current task.

Parametric design is not completely separated from traditional architectural design. It also relies on the orthodox tools used and combines them with the new ones like the huge computation capacity and formal freedom given by the computers. Architects not only use the cognitive model of traditional architecture when necessary during parametric design, a lot of tools of parametric design is also based on earlier solutions. The cause is that parametric design uses primarily mathematical and geometrical commands, and assists them with advanced programming solutions. These geometrical forms and transformations are often used for centuries in the architectural practice, but with the help of computers designers are able to use these tools in much higher number and with bigger efficiency.

The role and evolution of architecture during the history also highly influenced the tools used by architects highly. The introduced historical example represents the geometrical solutions mostly used on buildings before modernism. Out of necessity of quicker construction and the evolving automatization, the forms became simpler in the latest centuries, and because of the spirit of the age the asymmetry and irregularity were preferred above symmetry and regularity. Based on the topological point of view of parametric design it is possible to create buildings which form suits the contemporary demands, and uses all of the knowledge learned during the times. The usage of robotic manufacturing spreads worldwide which makes the creation of these freeform buildings possible.

3.5.2 NECESSARY KNOWLEDGE FOR PARAMETRIC DESIGN

All of this shows how widespread the necessary field of knowledge is for parametric designers. They have to know the traditional architectural design process and all of the typological knowledge, what it is based on. They have to be able to understand the geometrical connections and the topological build-up of the structures, which shows, that spatial skills are at least as important for parametric design, as it is for traditional architectural design. Parametric designers use parametric design thinking, and they also have to be able to use the parametric software. Finally, there are some theoretical knowledge from different scientific fields, mostly from mathematics and computer science, which is necessary to be able to apply and understand parametric design.

3.5.3 TOOLS OF TRADITIONAL AND PARAMETRIC ARCHITECTURE

In Figure 3.4. a simplified model of Mirror Palace is shown, which is located in the Amer Fort, located in Amer, Rajasthan, India. It was built in the 16th century. This example clearly represents the geometrical knowledge used for architecture before the time of computers. It shows how the architects were able to combine simple curves and surfaces with tangent curves and surfaces. They were also able to create the intersections of these surfaces. They used repetition and scaling to create the pattern in the cupola. These solutions are easily identifiable on a lot of European buildings too especially on churches and cathedrals.

Parametric architecture often uses the same tools and combines them with irregular patterns, freeform surfaces, and computation-heavy modifications. In Figure 3.5. the modification of a simple sphere

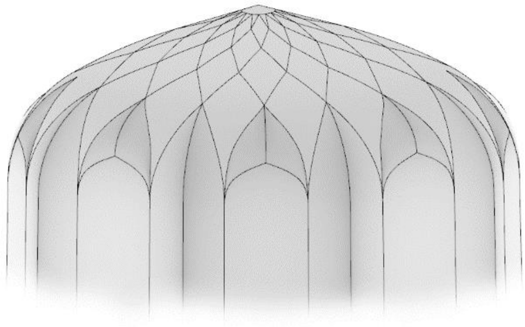


FIGURE 3.4. SIMPLIFIED MODEL OF MIRROR PALACE, AMER FORT, AMER, RAJASTHAN, INDIA

into a freeform surface is visible. This modification is based on the box you can see around the sphere and the new surface. This modification deforms the surface in the first box into the twisted box, which requires the computational capacity of a computer.

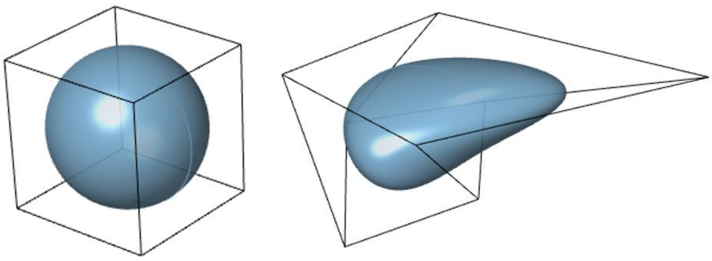


FIGURE 3.5. MODIFICATION OF A SPHERE WITH TWISTING ITS BOUNDING BOX

It is shown in Figure 3.6. that with parametric design tools it is possible to create the same regular pattern from the first example on this irregular surface. More detailed explanation about regular and irregular patterns is visible in *chapter 4*. On the third part of this image the model only contains flat surfaces.

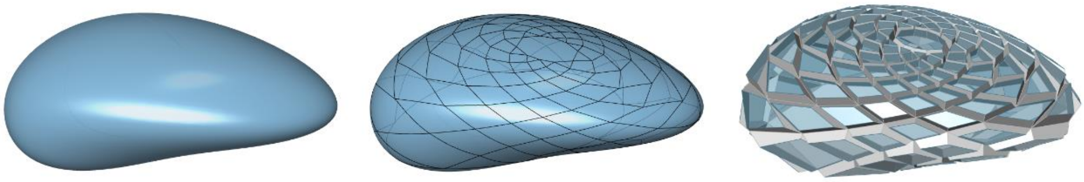


FIGURE 3.6. REGULAR PATTERN ON IRREGULAR SURFACE

In Figure 3.7. the same irregular surface is visible but in this case the pattern itself is irregular too. The last model in the figure is also made of only flat surfaces.

The next example shows another surface modification. With parametric tools designers are not only able to intersect surfaces, they are able to join and smooth them to create new freeform curved surfaces, as visible in figure 3.8. Figure 3.9 shows that the same pattern from the Amer Fort is also applicable on this surface.

FIGURE 3.8. SMOOTHING THE SURFACE WITH MESH TOOLS. MODEL INSPIRED BY VAULTED WILLOW BY MARC FORNES & THEVERYMANY IN BODEN PARK, EDMONTON, CANADA

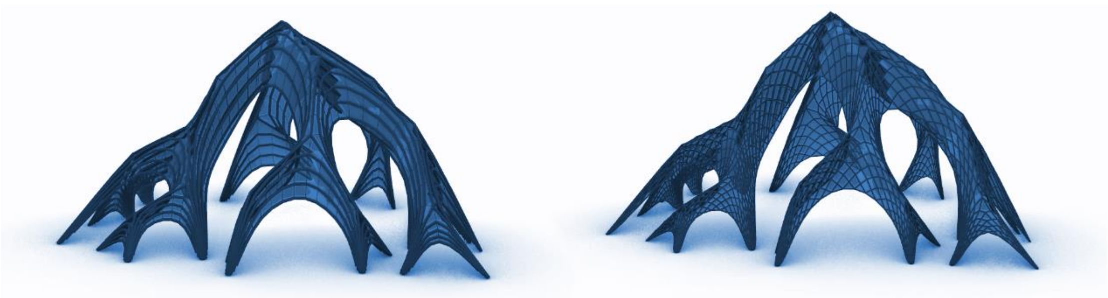


FIGURE 3.9. TWO DIFFERENT PATTERN ON THE SAME SURFACE

3.5.4 CONNECTION OF TRADITIONAL AND PARAMETRIC ARCHITECTURAL DESIGN

This overview of traditional and parametric design techniques represents that although the shapes created by parametric design tools are highly different from previously used architectural solutions, they lie on the same foundations. The evolution of the accessible design tools inducted the change in the architectural design process too. To follow this change acquiring new knowledge is necessary but the understanding of the traditional methods is also indispensable.

4 CLASSIFICATION OF PARAMETRIC DESIGN TECHNIQUES BASED ON THEIR GRAPH PATTERNS

4.1 INTRODUCTION

This classification is based on the mathematical representation of parametric design techniques. The patterns are studied based on their representation as graphs, and the classification system is made up to follow the logical steps of Parametric Design, also called as Parametric Design Thinking [14, 20, 21]. The aim of this classification system is to help designers to know and see through the production of the different shapes and patterns, make the understanding and learning of these structures easier, and help designers to understand the tools of parametric design easier. The topic of this chapter is the research of the architectural shapes, forms and patterns, and it does not include previously mentioned topics like urban and energy design. It hopefully it gives an overview of parametric design techniques and parametric design thinking.

In the first part of this chapter a new classification system will be introduced with its elements and evolution. After this the techniques in every category will be presented, accompanied with the application of these techniques on architectural examples. By the creation of the examples the software Rhinoceros [15] and Grasshopper [16] were used with other add-ons of Grasshopper.

4.2 EVOLUTION OF THE CLASSIFICATION SYSTEM

During the development of this classification system a lot of aspects of parametric techniques were researched based on many different resources [1-6], and the system went through a lot of change and refinement. First the planar and spatial patterns were separated, the planar patterns were researched based on geometrical aspects, and spatial patterns were classified based on structural solutions (for example: truss-grid structure, nexorade, tensegrity). This system gave a good starting point, but contained a lot of inconsistency, so it needed a harder reconsideration.

The main issue was the massive fragmentation of the system. It worked well with planar patterns, at the same time it was more of a listing of spatial patterns, then a classification of them. Even so the question which resulted to this system, came from the topic of planar patterns. Planar patterns were divided into two categories, tiling and subdivision. Tiling was defined as a pattern made of uniform elements. On many curved surfaces, as it will be represented later, it is not possible to create geometrically identical elements, even though the same logic is used for their creation. A graph based classification system was developed to solve this issue. This is also beneficial because it resulted a system which follows the steps of the parametric creation, and it is applicable on planar and spatial patterns too.

4.3 CLASSIFICATION SYSTEM

4.3.1 MAIN ATTRIBUTES OF THE CLASSIFICATION SYSTEM

The classification system can be characterized best with its three main attributes. These attributes are the following: 1) the researched structures, 2) steps of examination, 3) selection of the aspects of classification.

4.3.1.1 RESEARCHED STRUCTURES

In this work parametrically created structures were researched. Every tool available in a parametric design software is considered as parametric tool, even if it is as basic as rotation or multiplication or a complex command, made up various geometrical modifications. At the same time if a model can be easily created without parametric tools it is usually not considered as parametric structure. Based on this in this research those structures are investigated which can be only created with parametric tools, or the application of parametric tools makes the design process relevantly easier.

In this classification the structural aspect of a structure is not examined. The classification system is based on the representation of these structures and patterns as graphs. Using graphs to construe difficult models and patterns of the real world is a good working method, which can be seen in a wide range of scientific research [22-26].

As a consequence of this point of view the position of the vertices and the length of the edges is not an aspect in this classification, only the connection between the vertices, the topology of the pattern. The patterns are interpreted as undirected simple graphs, where both loops and multiple edges between the same two vertices are disallowed [27].

On the Figure 4.1. the representation of a pattern as a graph is visible.

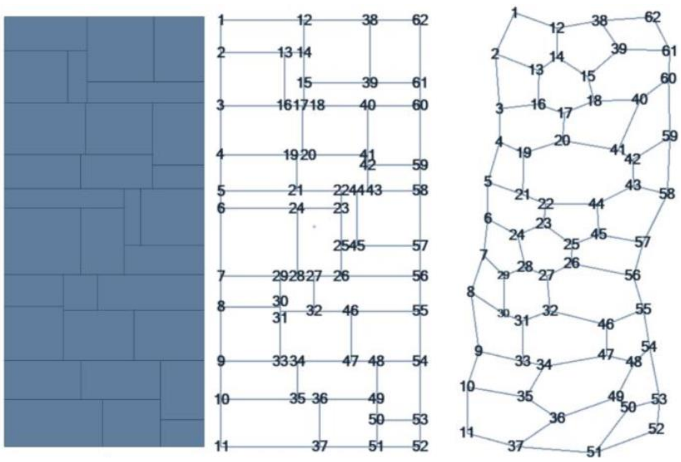


FIGURE 4.1. GRAPH REPRESENTATION OF A PATTERN

This point of view causes, that two patterns can be considered the same, even though they are created on different surfaces, planar or curved. On Figure 4.2. the same hexagonal pattern is visible on planar and on a hyperbolic surface. Although the hexagons are deformed on the hyperbolic surface, the graphs of the two patterns are close to be the same.

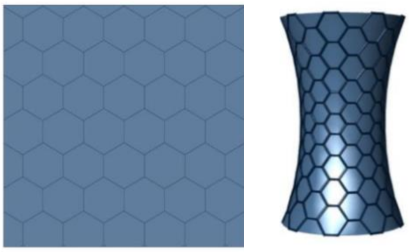


FIGURE 4.2. HEXAGONAL PATTERN ON PLANAR AND HYPERBOLIC SURFACE

4.3.1.2 STEPS OF EXAMINATION

The classification of the structures is based on the parametric workflow of their creation, as it can be seen in Figure 4.3. This makes the classification system practical and easily applicable.

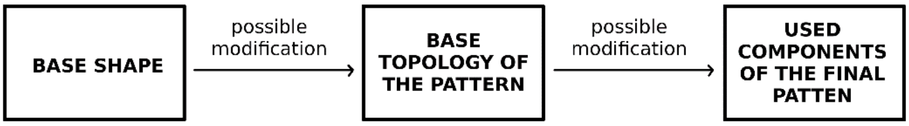


FIGURE 4.3. STEPS OF PARAMETRIC DESIGN

The examination follows the steps of the creation of a structure with parametric design techniques, so the aspects of the classification follows each other in the same order as they do during the creation of the pattern. During the creation it is possible to modify a structure without modifying its graph. These modifications will be mentioned later, and their practical application will be presented on examples.

In some cases, the classification can be made in two or more stages, which means, that the structure contains more patterns on different levels of their creation, which are more or less independent. This usually happens when a planar pattern is created a surface based on another pattern. The third example represents this “dual classification”.

4.3.1.3 SELECTION OF THE ASPECTS OF CLASSIFICATION

During the selection of the classification aspects the aim was to create categories which are clearly different, differentiable, and exact, and do not create useless categories. The aspects are always based on the topology or the modification of the topology of the pattern.

4.3.2 ASPECTS OF CLASSIFICATION

4.3.2.1 DIMENSION: PLANAR (ON-SURFACE) OR SPATIAL

The first aspect of the classification represents that the pattern is created on a planar or curved surface, or it is a spatial structure, as it can be seen in Figure 4.4. A pattern is called planar, if its topology is two dimensional, the interpretation of the pattern as a graph is a planar graph [27], where the graph can be drawn into a plane without its edges crossing each other.

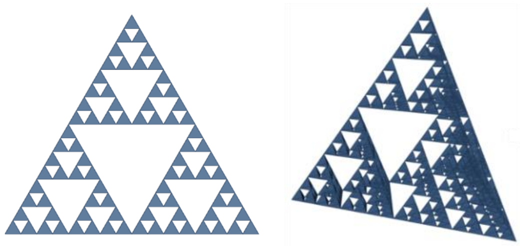


FIGURE 4.4. PLANAR AND SPATIAL VERSION OF THE SAME PATTERN

If the pattern is created on a surface, but it is not possible to create a planar graph, where the edges of the graph don't intersect each other, although they don't intersect each other on the initial surface, it is an exception of this rule. This exception is made in the case of surfaces with a genus bigger than 0 [28]. The genus of a surface equals to the number of the cutting curves of the surface which can cut the surface to be able to stretch into a plane.

An example is visible in Figure 4.5. The same hexagonal pattern is visible on a plane, on a cylinder, and on a torus. The pattern on the cylinder can be created by "sticking" the opposite edges of the plane together. By the same method the pattern on the torus can be created from the pattern on the cylinder. The graphs of the first two patterns are planar, but the third is not. The genus of the torus is 1. By the interpretation of its pattern as a graph, intersection points are created between non-intersecting edges. This exception allows to cut the pattern on the surface with as many curves as the genus of the surface, before creating the graph. This exception is made, because the creation and build-up of the represented patterns are the same, so they should appear in the same category.

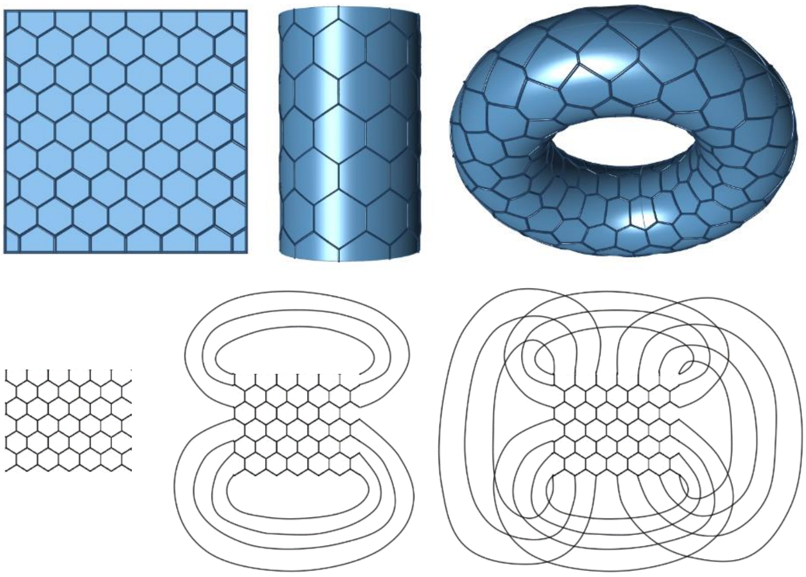


FIGURE 4.6. HEXAGONAL PATTERN ON SURFACES WITH GENUS 0 AND 1

The pattern visible in Figure 4.6. can be also considered as planar pattern, because the genus of the surface equals to the number of the triangles. Using the exception, it is possible to cut the pattern to get a planar graph. At the same time this example is a border-line case between planar and spatial patterns, and the way of the creation is the deciding factor.

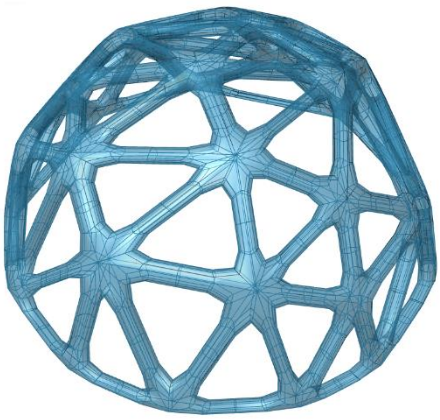


FIGURE 4.5. PATTERN ON A COMPLEX SURFACE

Between the planar and spatial patterns, the multi-layer patterns take place also as border-line cases.

A graph is called multi-layer graph, if the thickness [27] of it is bigger than 1, but they are made-up by the union of planar graphs. It means, that the multi-layer patterns are decomposable to the same number of planar graphs as the number of their thickness is. Since it is true to every non-planar graph, a pattern is only considered multi-layer pattern if the number of the layers is significantly lower than the number of the vertices and the pattern of the layers are connected to each other logically and can be derived from each other.

A multi-layer pattern can be seen in Figure 4.7.

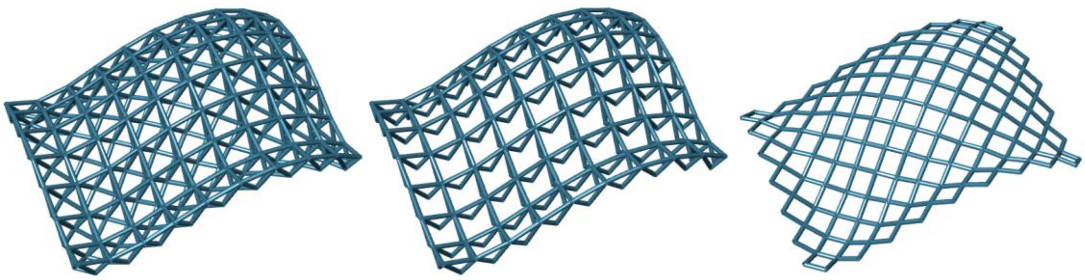


FIGURE 4.7. DOUBLE LAYER GRID (ON THE LEFT), AND ITS COMPONENTS (IN THE MIDDLE AND ON THE RIGHT)

Another special case is given by tree graphs [27]. Tree graphs are always planar graphs, because of their special topology. It is possible to take an exception, and consider some tree graphs as spatial creations, if they are created based on spatial structures. This means that the graph of the initial pattern itself was non-planar and the graph of the final structure is only planar because it does not contain all of the elements of initial pattern. For example, in Figure 4.8., the edges of the tree are created by using the edges of connecting pyramids.

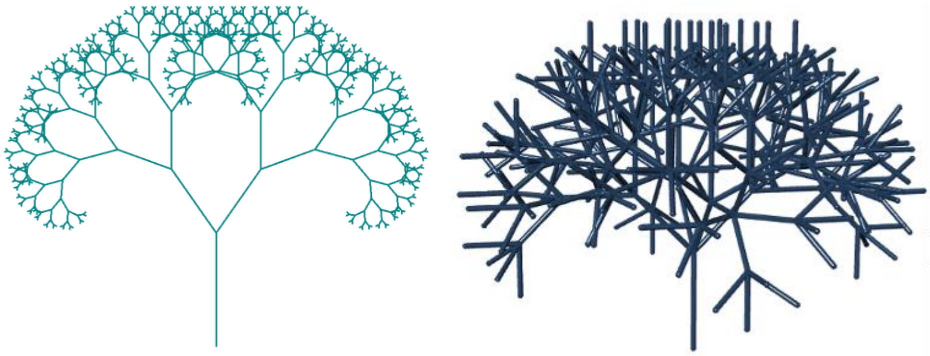


FIGURE 4.8. PLANAR AND SPATIAL TREE

4.3.2.2 TOPOLOGY OF THE PATTERN: REGULAR OR IRREGULAR

To separate the regular or irregular patterns the graph representation of these can be used to define a clear and exact solution. This is a universal classifying attribution of any pattern. The researched sources also deal with the part of the parametric design, which cover the tiling-subdivision-packing theme, because of its importance. In the work of Jane and Mark Burry [1] packing and tiling are mentioned and in the book of Jabi [2] both tiling, packing and subdivision are distinguished, where packing is defined as “the placement of many objects in space, in a way that little or nothing of it is left over" and subdivision is referred as a division of surfaces and generation of meshes in a way, that the results are suitable for Computer Numerical Control (CNC) machines. The concept of regular patterns in this classification system is based on tessellation, also called as tiling. In this classification a more generalized definition is used, based on graph topology, not

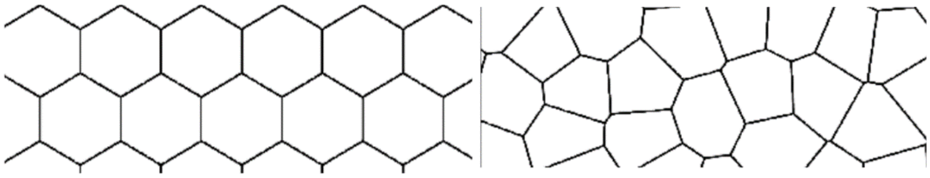


FIGURE 4.9. REGULAR AND IRREGULAR PATTERN

geometric shapes. An example for regular and irregular patterns can be seen in Figure 4.9. The graphs of planar patterns were first introduced, later it was expanded to the spatial patterns too, so they can be classified using the same rules.

The separation of regular and irregular patterns is based on the degree of their vertices [27], which is the number of the connecting edges of a vertex.

In this classification system some modifications of this definition are necessary based on practical issues. This rule has to be true only to the inner vertices of the pattern. If a regular geometric pattern is not infinite, the outer vertices which takes place on the margins of the pattern will create an irregular graph in graph theory, as it can be seen in Figure 4.10.

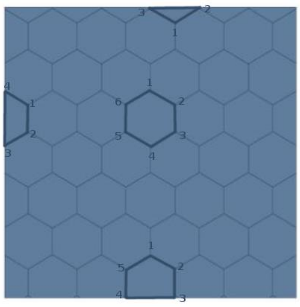


FIGURE 4.10. FACES IN HEXAGONAL PATTERN

A planar pattern is called irregular pattern, if – interpreted as a graph - one of the following is true:

- the number of neighbors of inside vertices varies;

or

- the number of vertices and edges of inner face varies;

where a face is a region bounded by the smallest possible continuous set of edges, where each member participates only once and the point of origin is identical to the point of arrival.

A planar pattern is called regular pattern, if – interpreted as a graph – the following conditions are both true:

- the number of neighbors of inside vertices are equal;

and

- the number of vertices and edges of inner faces are equal.

The side vertices and edges of the graphs do not have to obey this law.

In the case of planar patterns, the following definition is equal to the previous: the pattern is called regular, if the graph of the pattern and the dual graph [28] of it are both regular graphs, in the case of the inner vertices, as it can be seen in Figure 4.11. To create the dual graph of a graph, all of the faces of the initial graph have to be replaced with a vertex, and if two face have a common edge, the corresponding vertices will be neighbors.

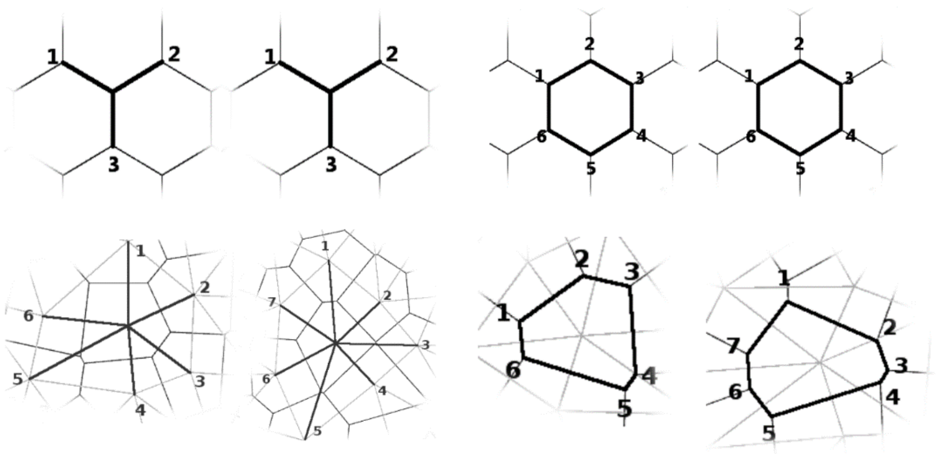


FIGURE 4.11. NEIGHBOURING VERTICES AND FACES IN REGULAR AND IRREGULAR PATTERNS

The first definition can't be applied on patterns with tree graphs, because they do not contain any faces, as it can be seen in Figure 4.12., so a pattern with a tree graph is called regular, if it fulfills the second definition.

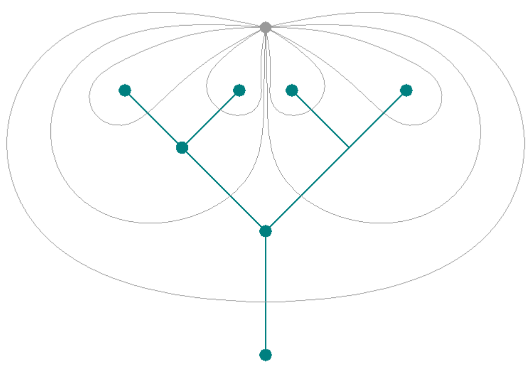


FIGURE 4.12. DUAL GRAPH OF A TREE GRAPH

In case of spatial patterns, the cells also have to suffice the same rule as faces to be considered as regular pattern. So a spatial pattern is regular if:

- the number of neighbors of inside vertices are equal;
and
- the number of vertices and edges of inner faces are equal;
and
- the number of vertices and edges and faces of inner cells are equal.

If a pattern does not fulfill these rules it is considered as irregular pattern.

A pattern is called semi-regular if the degree of the vertices or the faces – or cells in the case of spatial patterns – is not the same, but it is made up of finite number repeating elements with the same degree, like Archimedean tilings, also called as semi-regular tessellations [17], as it can be seen in Figure 4.13.

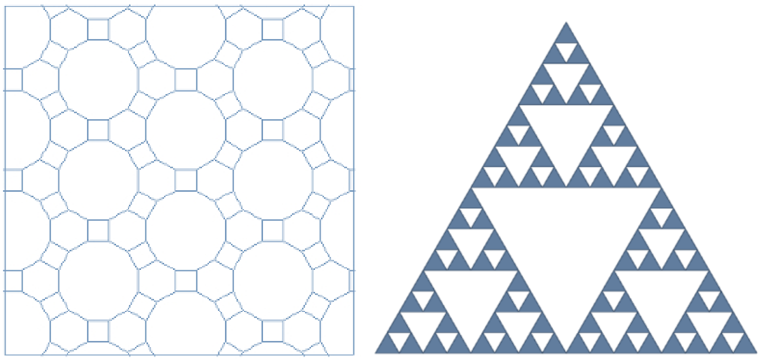


FIGURE 4.13. SEMI-REGULAR PATTERNS

Fractals can be also considered as semi-regular patterns, because they are created by a repetitive or recursive sequence of instructions, as it can be seen in figure 4.13.

The creation of these semi-regular patterns is very similar to the creation of the regular patterns and fundamentally different to the creation of the irregular patterns, so this category is considered a subcategory of regular patterns.

4.3.2.3 COMPONENTS: VERTICES, EDGES, FACES, CELLS, OR COMBINED

This aspect of the classification represents the materialization of the topology, it shows what kind of elements make up the structure. The classification is based on primarily on the topology of the pattern, not the exact geometry. It can be hard to decide if an element is a thick face or a cell if only the geometry of it is taken under consideration. Because of this it is more practical to investigate, what part of the pattern was used to create the structure, as it can be seen in Figure 4.14. Although it is relatively rare to find an architectural structure made by the vertices of the pattern, but it is possible to create one, so it is part of the classification system. It is possible to use more topological parts to create the structure, in this case it is called combined structure.

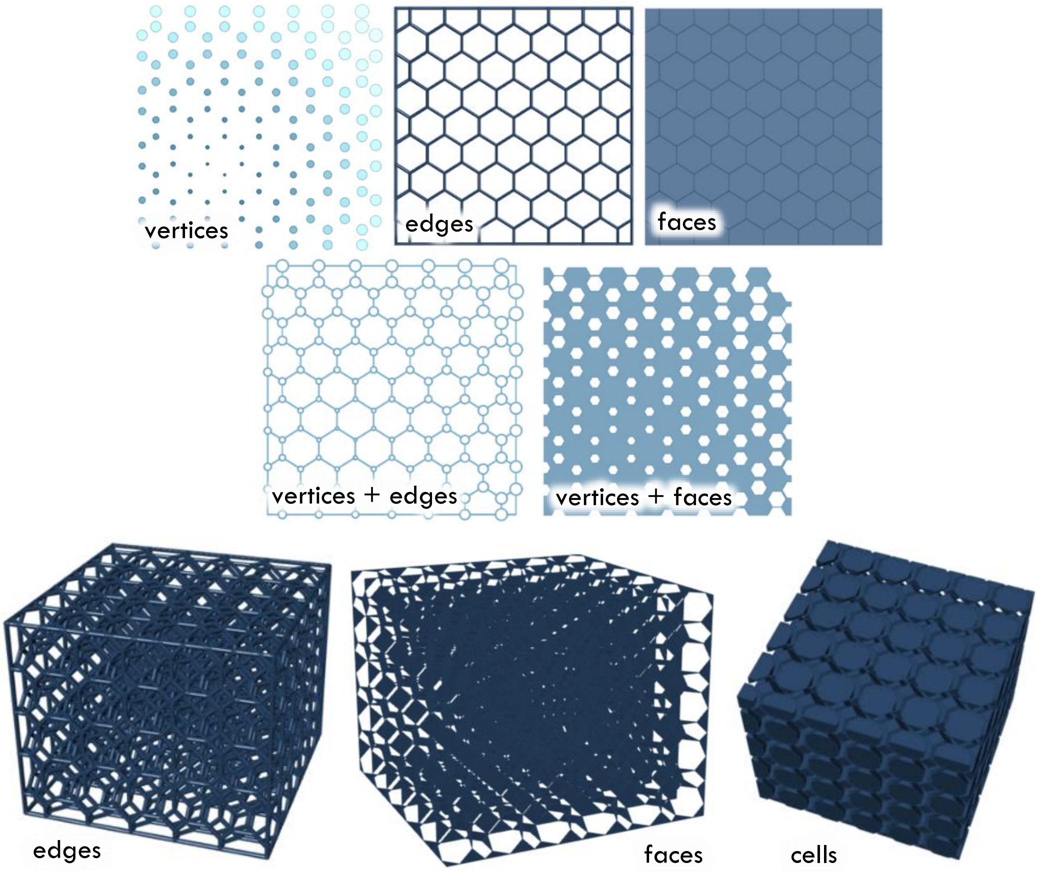


FIGURE 4.14. PLANAR AND SPATIAL PATTERNS WITH DIFFEENT COMPONENTS

4.3.3 OTHER POTENTIAL ASPECTS TO EXAMINE

During the creation of the classification system more possible aspects came to consideration. The following three of them were finally not included into the system, but there are worthy to mention. These aspects are mostly sensible on planar patterns and less on spatial patterns. This is another reason why these are not part of the final classification system.

4.3.3.1 SHAPE OF SURFACES

In the case of planar patterns, the shape of the surface on which they were created can be a logical part of the classification system. It is easy to involve this aspect to the classification process of planar patterns, and every planar pattern can be classified by this aspect.

Surfaces by their shapes can be separated into planar or curved surfaces. The curved surfaces can be curved in the direction of their axis k_1 and k_2 – axis x and axis y in Figure 4.15. –, these are called the principal curvatures [29] of the surface. The Gaussian curvature [29] is the product of these two values, and this indicates the type of the curved surfaces, as it can be seen in Figure 4.15.

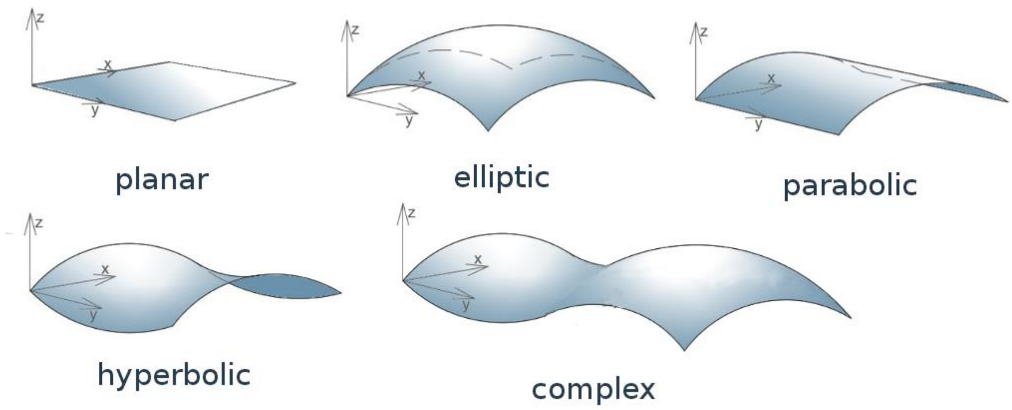


FIGURE 4.15. SURFACE TYPES

Surfaces can be differentiated by the creation process of them, like extrusion, translational, rotational, ruled or freeform surfaces [17].

The differentiation of the surface patterns by their surface type would create a lot of new categories, but it would not create relevant change in the classification system.

4.3.3.2 FULL OR LACUNARY PATTERN

Another interesting aspect is, if the planar pattern covers the surface on which it was created on fully on not, as it can be seen in Figure 4.16. The creation of the pattern is the same in both cases, but in the case of lacunary patterns not all of the components are used or some parts of it is removed. Because of this it is not an actual classification aspect, more like a modification of a pattern.

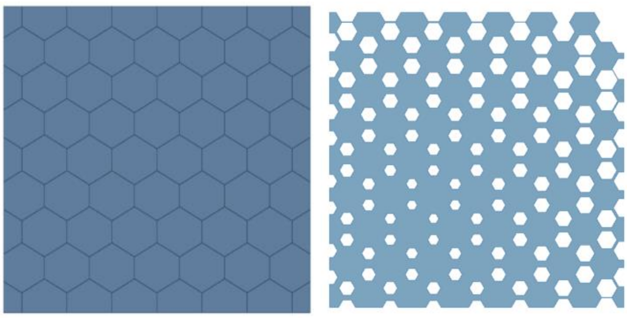


FIGURE 4.16. FULL AND LACUNARY PATTERN

4.3.3.3 ON SURFACE OR SURFACE MODIFYING PATTERN

When the creation of a pattern is based on a surface, in the last steps the pattern is sometimes distanced from the surface slightly. By moving the vertices out of the surface it is possible to create weaved patterns, as it can be seen in Figure 4.17., or reciprocal structures, also called as nexorades [30], and a lot of interesting scale-like patterns. Although the topology of these patterns are planar, the structure no longer lays in the surface.

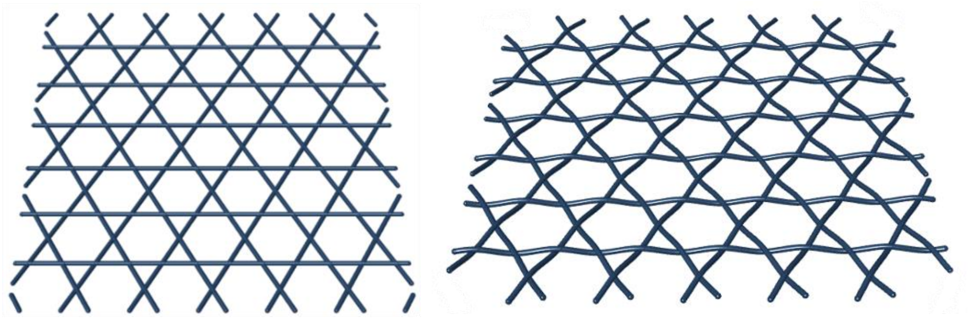


FIGURE 4.17. ON SURFACE AND SURFACE MODIFYING PATTERN

4.4 THE CLASSIFICATION SYSTEM, AND THE CLASSIFIED STRUCTURES

4.4.1 CLASSIFICATION SYSTEM AS A SCHEDULE

Based on the previously introduced classification aspects there are 2*2*5=20 categories in this system. All of these categories are visible in Table 4.1. For an easier manageable system, the properties are represented by numbers in the positional notations of the aspects. In this system the 1.1.4 and 1.2.4 categories do not exist, because it is not possible to create a surface pattern with cell components. The spots are signed with gray color in the Table 4.2., and it results in 18 categories finally.

<table>
<tr><td>I</td><td colspan="3">PLANAR (1)</td><td colspan="2">SPATIAL (2)</td></tr>
<tr><td>II</td><td colspan="3">REGULAR (1)</td><td colspan="2">IRREGULAR (2)</td></tr>
<tr><td>III</td><td>VERTICES (1)</td><td>EDGES (2)</td><td>FACES (3)</td><td>CELLS (4)</td><td>COMBINED (5)</td></tr>
</table>

TABLE 4.1. CLASSIFICATION SCHEDULE

	I. 1-2	II. 1-2	IV. 1-5
1.	1	1	1
2.	1	1	2
3.	1	1	3
	1	1	4
4.	1	1	5
5.	1	2	1
6.	1	2	2
7.	1	2	3
	1	2	4
8.	1	2	5

	I. 1-2	II. 1-2	IV. 1-5
9.	2	1	1
10.	2	1	2
11.	2	1	3
12.	2	1	4
13.	2	1	5
14.	2	2	1
15.	2	2	2
16.	2	2	3
17.	2	2	4
18.	2	2	5

TABLE 4.2. CLASSIFICATION CATEGORIES

4.4.2 CLASSIFICATION SYSTEM AS A FLOW DIAGRAM

In the diagram in Figure 4.18. it is possible to follow the steps of the classification to discover the tree of the categories and select the paths for the patterns. The flow diagram in Figure 4.19. also contains examples of the classified structures.

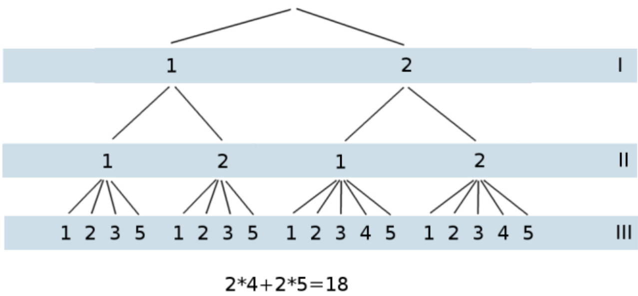


FIGURE 4.18. DIAGRAM OF THE CLASSIFICATION SYSTEM

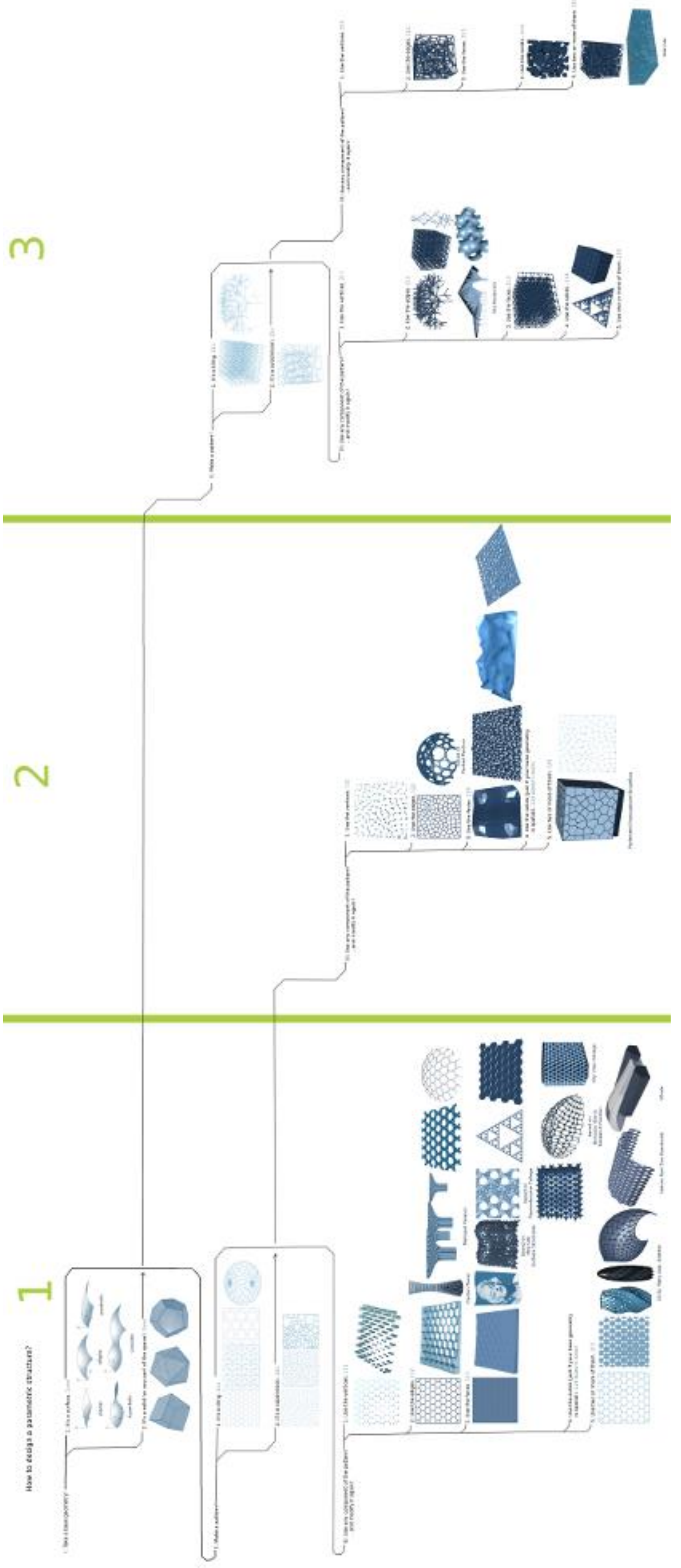
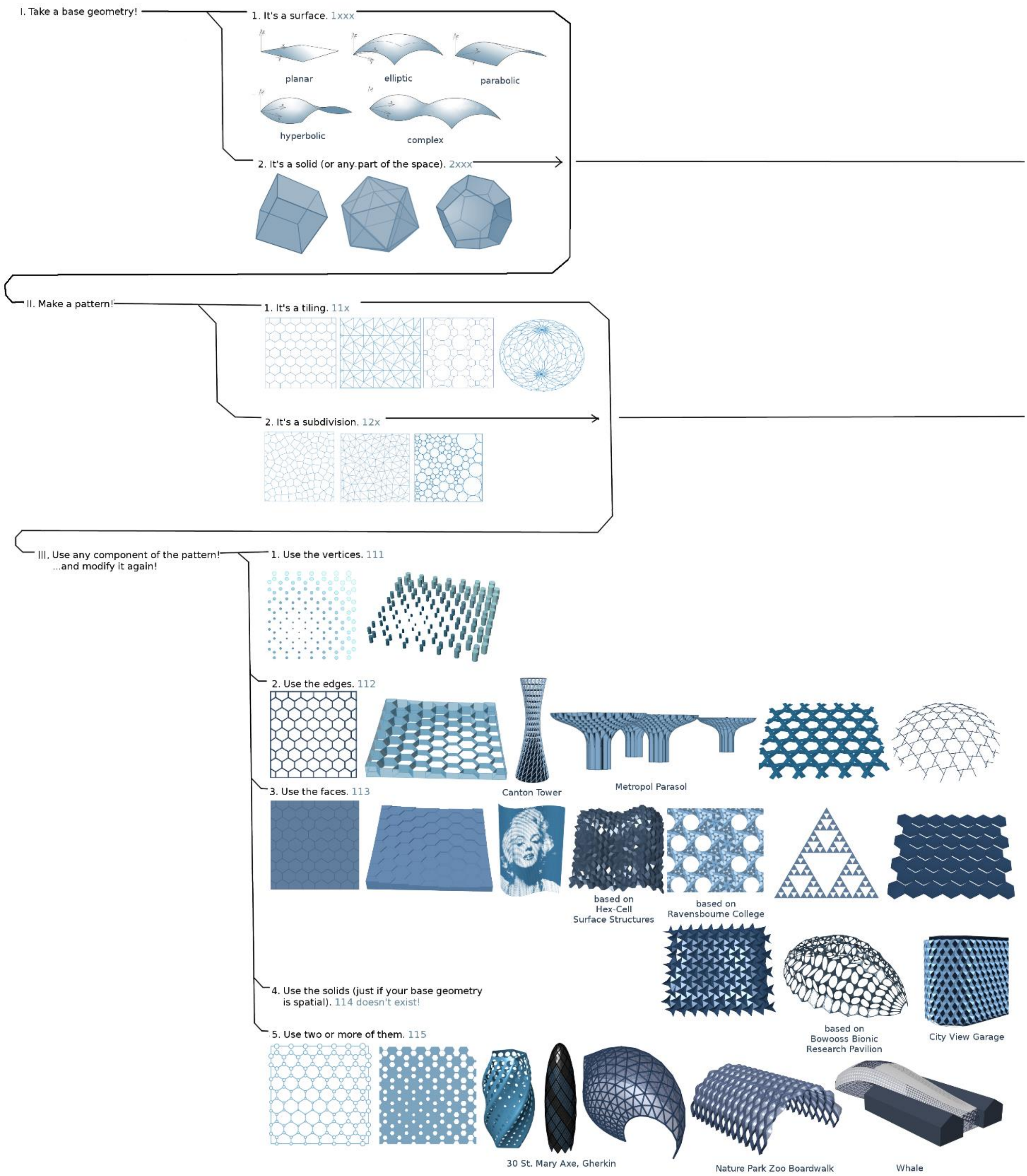
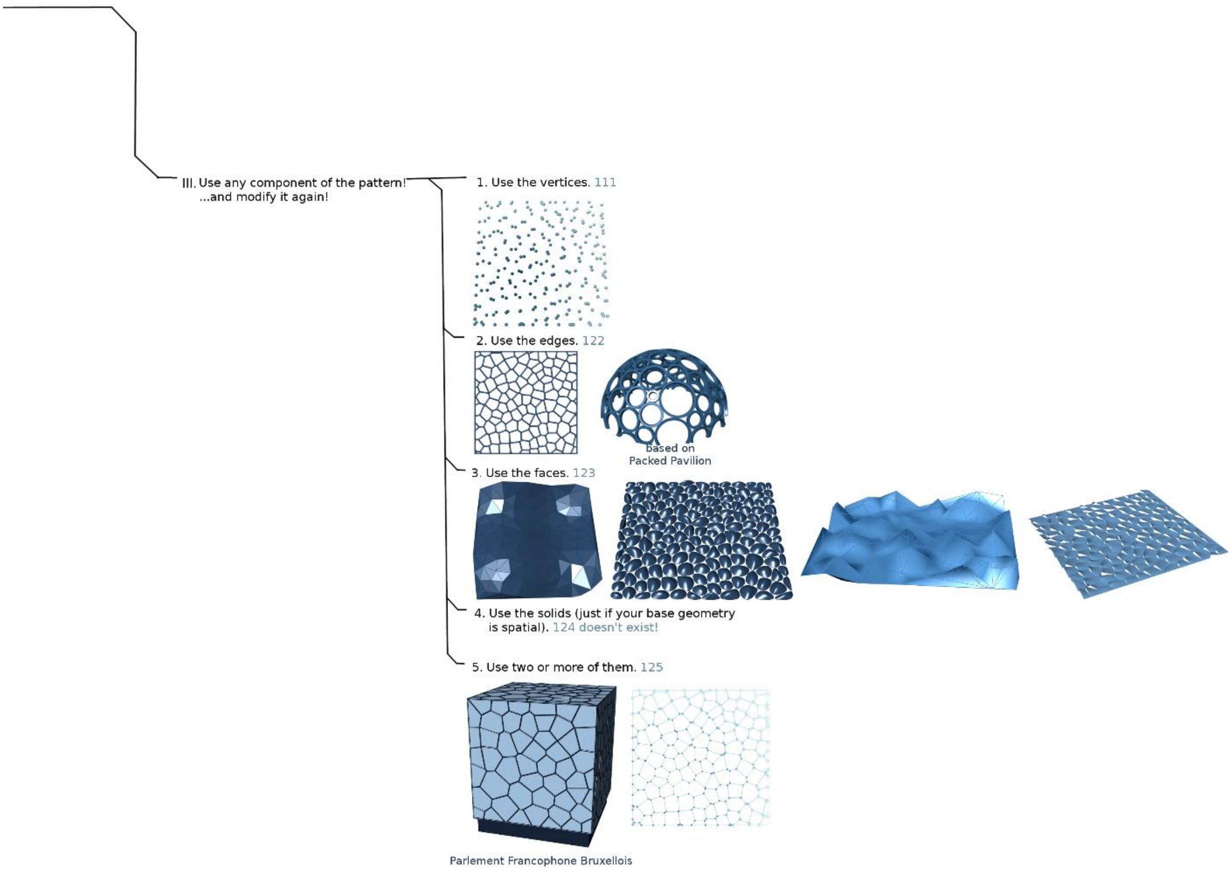


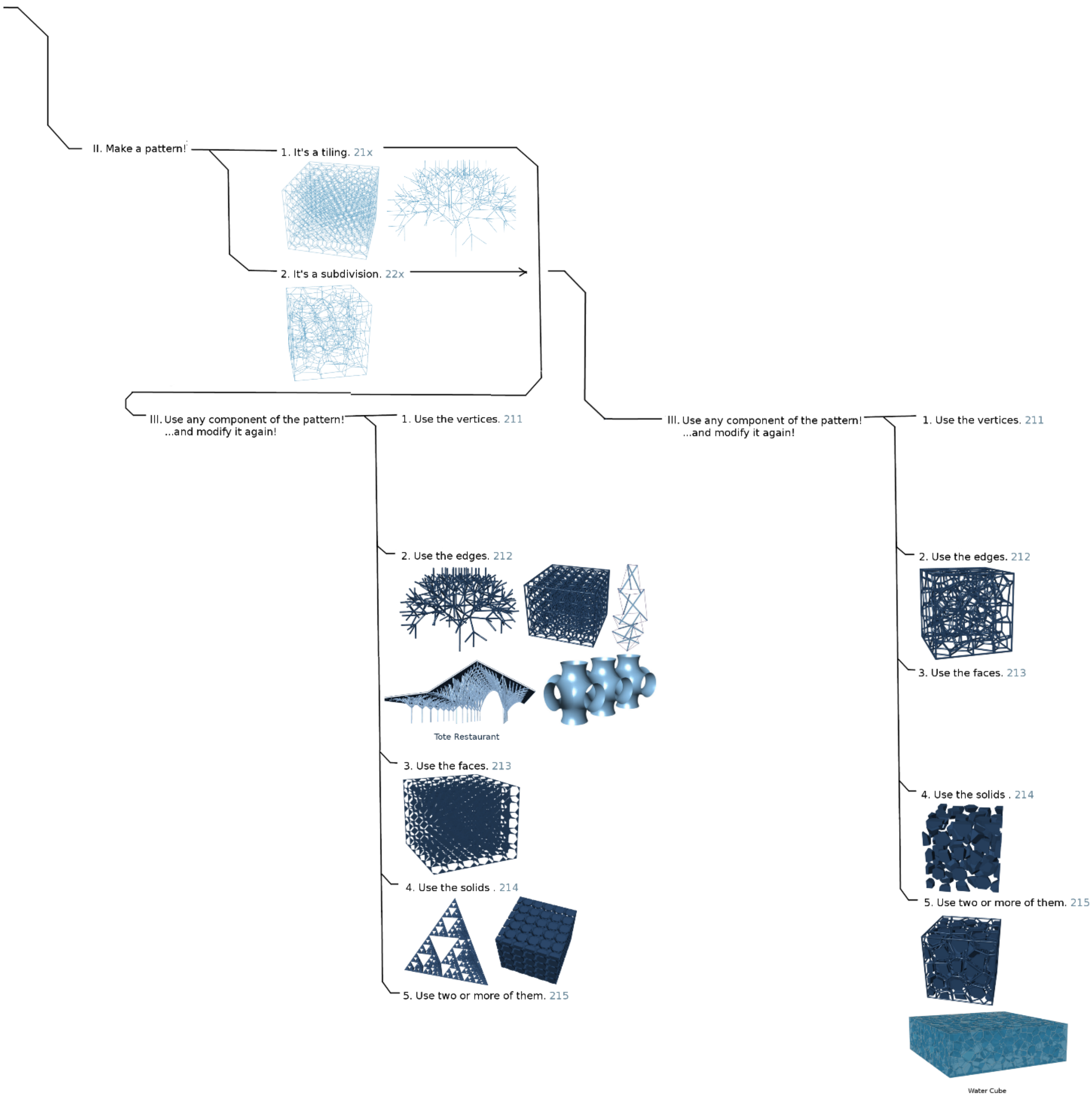
FIGURE 4.19. FLOW DIAGRAM OF THE CLASSIFICATION SYSTEM WITH EXAMPLES. BIGGER VERSION ON THE NEXT PAGES

How to design a parametric structure?





III. Use any component of the pattern!
...and modify it again!
1. Use the vertices. 111
2. Use the edges. 122
based on
Packed Pavilion
3. Use the faces. 123
4. Use the solids (just if your base geometry
is spatial). 124 doesn't exist!
5. Use two or more of them. 125
Parlement Francophone Bruxellois


II. Make a pattern!
1. It's a tiling. 21x
2. It's a subdivision. 22x
III. Use any component of the pattern!
...and modify it again!
1. Use the vertices. 211
2. Use the edges. 212
Tote Restaurant
3. Use the faces. 213
4. Use the solids . 214
5. Use two or more of them. 215
III. Use any component of the pattern!
...and modify it again!
1. Use the vertices. 211
2. Use the edges. 212
3. Use the faces. 213
4. Use the solids . 214
5. Use two or more of them. 215
Water Cube

4.5 EXAMPLES FOR THE CLASSIFICATION

4.5.1 EXAMPLE FOR PLANAR PATTERN – VASE

Figure 4.20. shows an example for planar patterns. It is not an architectural structure, but it represents the creation and also the classification method of surface patterns clearly. The surface is created first with the multiplication and rotation of the base polygon. In the next step the surface is divided with a tetragonal net, which follows the rotation of the surface. By the vertices of the pattern circles are created with varying radius. It is easy to see in Figure 4.20. that the pattern is a surface pattern with a regular pattern and vertex components.

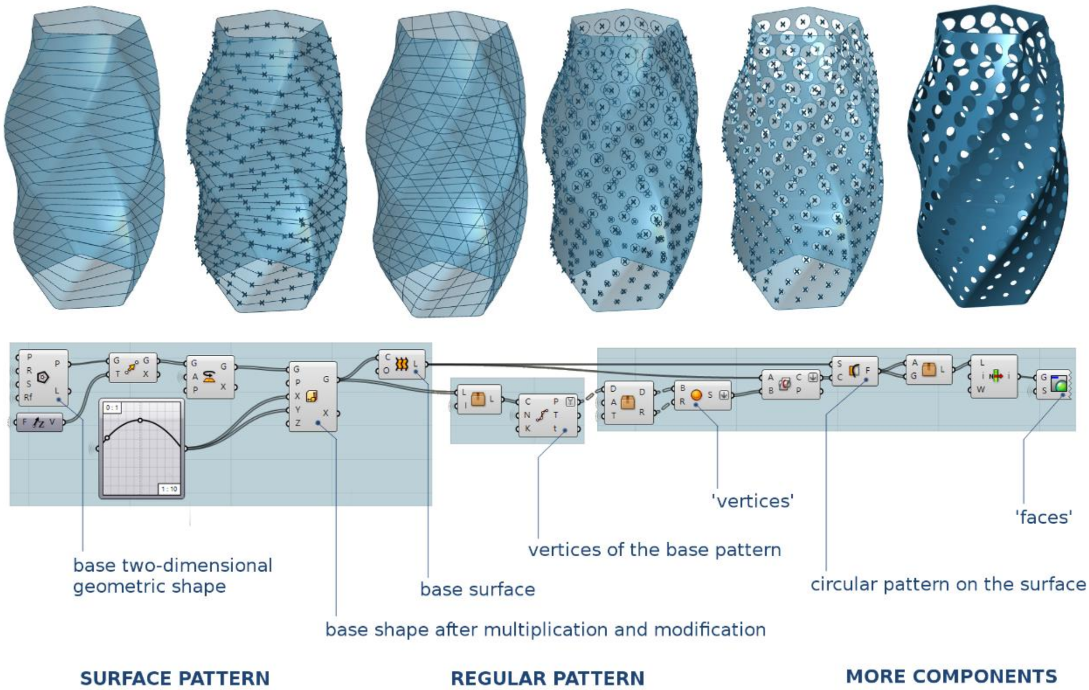


FIGURE 4.20. VASE MODEL AND ITS GRASSHOPPER DEFINITION

4.5.2 EXAMPLE FOR SPATIAL PATTERN – WATER CUBE

This example can be seen in Figure 4.21., and it is based on the schematic model of the Beijing National Aquatics Center [31]. In the first step the base form – the walls of the building – is created from cuboids. The Voronoi pattern is made from randomly created points. The random position of the initial points results random Voronoi cells with varying degree of the vertices, which is an irregular pattern. The edges are used to create the beams of the structure, and the light transmitting building envelope is created from the faces of the cells. This means that the pattern is a spatial irregular pattern with edge and face components.

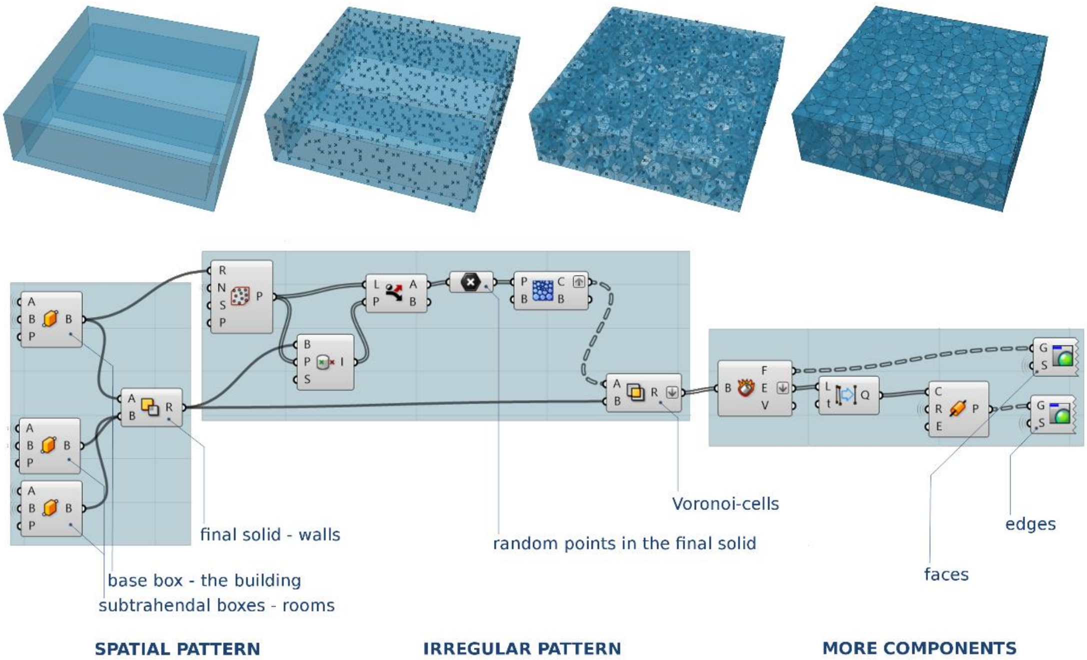


FIGURE 4.21. SIMPIFIED MODEL OF THE BUILDING AND ITS GRSSHOPPER DEFINITION

4.5.3 EXAMPLE FOR DUAL CLASSIFICATION

In Figure 4.22. the structure is created and also classified in two steps, because on the surface of the first pattern another pattern is created. In the first step random points are created on the surface of a half sphere and connected into a triangular grid with different vertex degree. The structure is created from the edges of the triangular pattern. On the surface of it a new pattern is created. It is a regular tetragonal pattern, and finally the edges of this pattern are used. The first pattern is an irregular surface pattern with edge components, the next one is a regular surface pattern also with edge components.

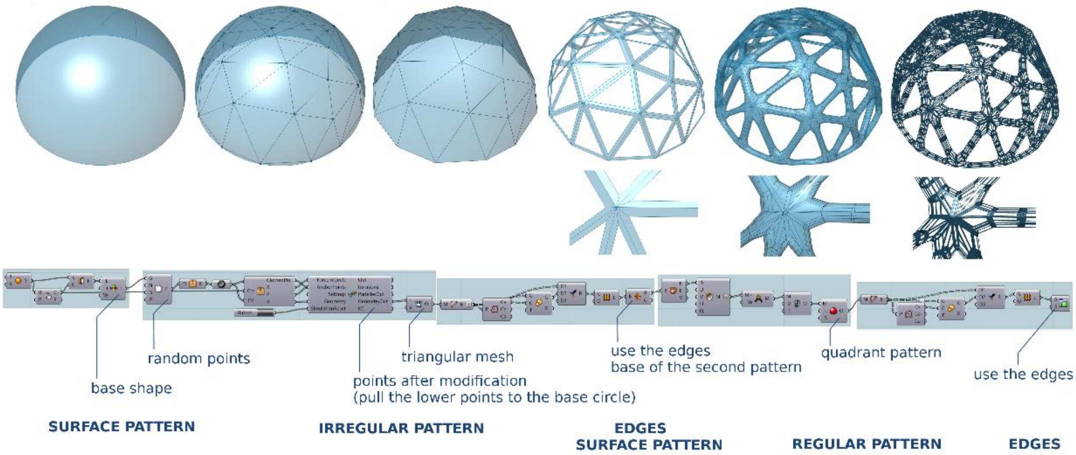


FIGURE 4.22. MODEL OF THE DOME AND ITS GRASSHOPPER DEFINITION

4.5.4 EXAMPLE BUILDINGS

In Table 4.3. some architectural examples can be seen, accompanied with their classification based on the previously presented system.

Building	Planar / Spatial	Regular / Irregular	Used elements	Other aspects
Lincoln Park Zoo Pavilion, Chicago, USA, Studio Gang Architects	Planar	Regular	Faces Edges	Surface modifying,
30 St Mary Axe, (The Gherkin), London, UK, Foster and Partners	Planar	Regular	Faces	
Whale Budapest, Hungary, Kas Oosterhuis	Planar	Regular	Faces	Lacunary
Vanke Pavilion, Milan, Italy, Daniel Libeskind	Planar	Regular	Faces	Surface modifying
Origami Building, Paris, France, Manuelle Gautrand	Planar	Regular	Faces	Surface modifying, Lacunary
Al Bahar Towers, Abu Dhabi, EAE, Aedas Architects	Planar	Regular	Faces	Surface modifying, Lacunary, Kinetic
Bowoos Bionic Research Pavilion, Saarbrucken, Germany, 2012, Students of Saarland University	Planar	Regular	Faces	Surface modifying, Lacunary
Miami Design District City View Garage, Miami, FL, USA, Iwamottoscott Architecture	Planar	Regular	Faces	Surface modifying, Lacunary
The Grand Egyptian Museum, Giza, Egypt, Heneghan Peng Architects	Planar	Regular (Semiregular)	Faces	Fractal

Ravensbourne College of Design and Communication, London, UK, Foreign Office Architects	Planar	Regular (Semiregular)	Faces	
Dermoid at Convergence, Melbourne, Australia, 2013, SIAL RMIT, CITA	Planar	Regular (Semiregular)	Edges	Surface modifying, Nexorade
Aragn Pavilion, Zaragoza, Spain, 2008, Daniel Olano, Mendo Architects	Planar	Regular Irregular (at the ends)	Faces Edges	Surface modifying, Lacunary
Supertree Grove, Singapore, Grant Associates	Planar	Irregular	Edges	
Packed - a cardboard pavilion, Exhibition at Arts & Crafts Museum Shanghai and Fudan University Shanghai, Min-Chieh Chen, Michele Leidi, Dominik Zausinger	Planar	Irregular	Edges	Lacunary
Metropol Parasol, Seville, Spain, Jürgen-Meyer Hermann	Spatial	Regular	Faces	
NonLin/Lin Pavilion, Orleans, France, 2011, Marc Fornes	Spatial	Irregular	Edges	Dual classification (additional surface pattern)
Beijing National Aquatics Center, Beijing, China, PTW Architects, CSCEC, CCDI, Arup	Spatial	Irregular	Cells	
Acme UN Memorial Building, Older & Buzzing	Spatial	Irregular	Cells	
Plasti(K) Pavilion, St-Louis, MO, USA, 2011, Marc Fornes	Spatial	Irregular	Cells	Lacunary

TABLE 4.3. ARCHITECTURAL EXAMPLES AMD THEIR CLASSIFICATION

4.6 CONCLUSION

The representation of patterns as graphs helped to construct a clear and simple to use topological classification for patterns of parametric design techniques. This classification with minor modifications can be applied on spatial structures too. Because this classification system is based on the same logical steps as the creation of the patterns and structures, this can help designers to understand parametric design better.

5 FORMEX ALGEBRA ADAPTATION INTO PARAMETRIC DESIGN TOOLS

5.1 INTRODUCTION

This chapter describes the adaptation of the Formex configuration processing to the computer program Grasshopper 3D [16]. It covers the shapes of dome and vault structures of Formex Configuration Processing I [32], and Formex Configuration Processing II [33]. For example, it includes the 'simple' dome structures like Ribbed domes, Schwedler domes and Lamella domes, which are based on spherical coordinates. The vault and planar structures based on cylindrical coordinates, and the triangular version of these structures based on the Diamatic domes are also introduced. These structures are visible in the Figure 5.1.

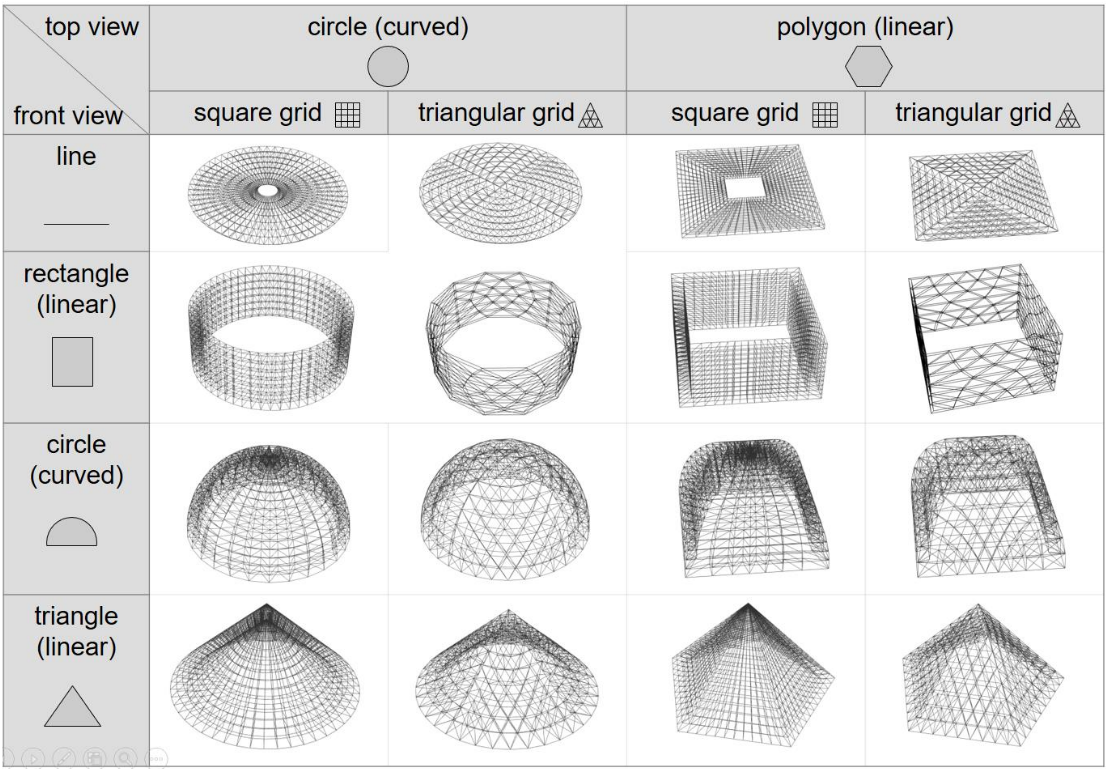


FIGURE 5.1. ALL TYPE OF DOME AND VAULT TOOLS

The chapter also introduces the generation of two additional shapes based on a newly derived formulation. These shapes are not part of Formex configuration processing but they use the possibilities of parametric modeling and fit into this system.

Using computer-based technologies to plan truss grid structures provides the possibility for architects and engineers to use more advanced or user-friendlier methods in the design process [34, 35]. The aim of this adaptation is to develop a tool for architects to design grid structures easily. In this chapter the generation methods of Formex algebra are reinterpreted and different calculation methods are used.

Formex algebra is a mathematical system created by Hoshyar Nooshin and Peter Disney in 1975 [32]. It is developed since then by Professor Nooshin and his colleagues [36]. It is primarily used for planning various truss-grid structures, mainly domes and vaults, but other geometrical forms are also covered by it [37]. Formex algebra can be used with the computer program Formian (Formian 2 and Formian K. [38]), and the programming language Formian.

Formex algebra uses coordinate system transformations to modify grid structures. This solution makes very easy to construct a vault or a dome from a planar or multi-layered grid, which is designed in the Cartesian coordinate-system. At the same time, it is necessary to learn the programming language Formian to be able to use Formex algebra, which makes its application difficult.

5.2 DOME STRUCTURES

5.2.1 MATHEMATIC CALCULATIONS

The calculation of dome structures is based on spherical coordinates. In Formex algebra the user has to define three variables, *b1* (factor for scaling in the first direction (linear scale factor)), *b2* (factor for scaling in the second direction (angular scale factor)), and *b3* (factor for scaling in the third direction (angular scale factor)) [32]. To change any other attribute of the dome structure, the user has to change the properties of the initial grid. The thickness of the double layer grids is the same as the thickness of the initial grid and the gap in the middle is defined by the position of the initial grid. This solution requires only a few variables, but the user has to calculate the necessary measurements.

By the reformulation of the dome tool a versatile and user-friendly solution could be created. The original calculation method is modified and extended by using an ellipsoidal coordinate system. In this way wider range of dome shapes can be created, for example ellipsoidal domes with different height and radius, compared to a simple spherical solution, can be seen in Figure 5.2.

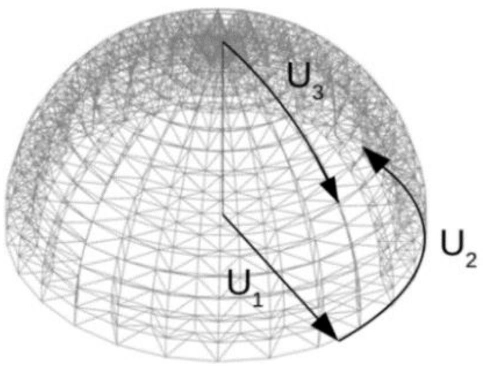


FIGURE 5.2. SPHERICAL COORDINATES

The coordinates of the spherical coordinate system are calculated from the original coordinates of the initial grid, see in Figure 5.3.

$$U_1 = R \quad (6.1)$$
$$U_2 = \alpha * x$$
$$U_3 = \beta * y$$

where

- U_1, U_2, U_3 are the coordinates of the spherical coordinate system;
- x, y, z are the Cartesian coordinates of the initial grid;
- R is the radius of the spherical dome;
- and α, β are the factors of angular scaling in the second and third directions.

The calculation of the ellipsoidal coordinates is similar to the spherical coordinates, only the first coordinate (U_1) of the points vary according to the spherical coordinates of a point, as it can be seen in Figure 5.4., the calculation of U_2 and U_3 are the same.

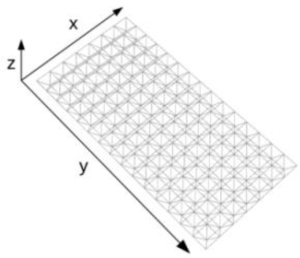


FIGURE 5.3. INITIAL GRID

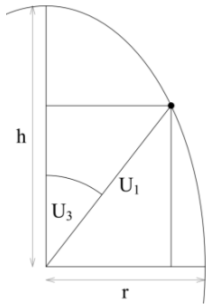


FIGURE 5.4. VARIABLES OF THE ELLIPSOIDAL COORDINATES

$$U_1 = \sqrt{\frac{1}{\frac{[\sin(U_3)]^2}{r^2} + \frac{[\cos(U_3)]^2}{h^2}}} \quad (6.2)$$

where

- r is the radius of the base of the ellipsoidal dome;
- and h is the height of the ellipsoidal dome.

In the new formulation of the dome tool no scaling factor is used. The first coordinate (U_1) is calculated from the radius and height variables. Because of this the thickness of the double layer grid is also a required variable, which also changes the calculation of U_1.

To ensure the uniformity of the ellipsoidal structure, the inner shell of the grid is calculated by an ellipsoid, where the height and the radius are reduced by the value of the thickness, as it can be seen in Figure 5.5.,

$$U_{1i} = \sqrt{\frac{1}{\frac{[\sin(U_3)]^2}{(r-t)^2}+\frac{[\cos(U_3)]^2}{(h-t)^2}}} \qquad (6.3)$$

where

- U_{1i} is the first coordinate of the inner shell;
- and t is the thickness of the structure.

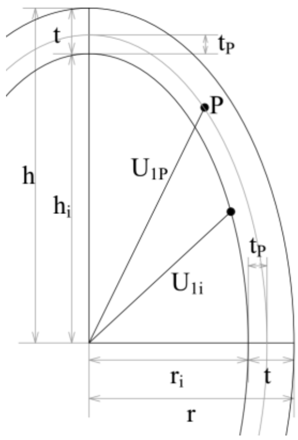


FIGURE 5.5. VARIABLES OF ELLIPSOIDAL COORDINATES WITH THICKNESS

According to this, the calculation of the first coordinate of a point is visible in (6.4)

$$t_P = t\frac{|z_P - z_{min}|}{|z_{max} - z_{min}|} \qquad (6.4)$$

$$U_{1P} = \sqrt{\frac{1}{\frac{[\sin(U_{3P})]^2}{(r-t_P)^2}+\frac{[\cos(U_{3P})]^2}{(h-t_P)^2}}}$$

where

- U_{1P}, U_{2P}, U_{3P} are the ellipsoidal coordinates of a point in the grid;
- t *is* the distance of a point from the inner shell;
- z_P is the z coordinate of a point in the initial grid;
- and z_{max}, z_{min} are the largest and smallest z coordinates in the initial grid.

In Formex algebra the gap in the center of the structure is specified by the position of the initial grid according to the origin. In the new calculation method, a variable is used to define the angle of the gap. If the dome structure has a gap, the calculation of the U_3 varies as it is plotted in Figure 5.6.

$$U_{3P} = \lambda + \beta * y_P \qquad (6.5)$$

where

- λ is the angle of the gap;
- and y_P is the y coordinate of a point in the initial grid.

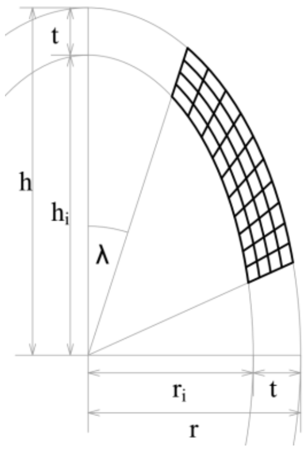


FIGURE 5.6. VARIABLES OF AN ELLIPSOIDAL DOME WITH A GAP

5.2.2 MODIFIED DOME STRUCTURES

With the modification of the reformulated dome tool some additional shapes can be created with which users can construct various ellipsoidal coordinate system based shapes. This modification adds a lot of new opportunity for shape definition to this tool.

In these modified structures one or two of the angular coordinates of the dome structure are modified and replaced with linear coordinates. The outer vertices of the transformed grid remain in place, but one, two or three edges of the structure are transformed from circular to linear. During this transformation the ratio of the points along the edges remain the same, so the division of the grid will remain equal, if it was equal in the initial grid. In Figure 5.7. the three modification types can be seen. The modified structure is called cupola if the second coordinates are transformed from angular to linear (Figure 5.7. *a*), cone, if the third coordinates are transformed (Figure 5.7 *b*), and pyramid if both the second and the third coordinates are transformed from angular to linear (Figure 5.7 *c*).

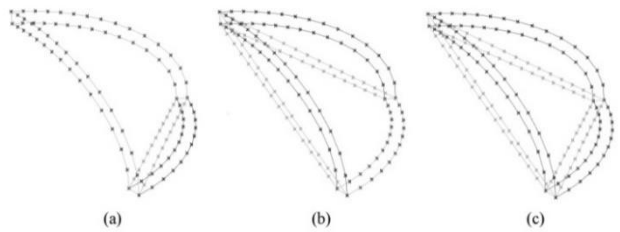


FIGURE 5.7. EDGES OF THE MODIFIED DOMES (A) TYPE 1 - CUPOLA, (B) TYPE 2 - CONE, (3) TYPE 3 - PYRAMID

These tools were created to fit into the ellipsoidal coordinate system. The thickness of the structures by the minimal and maximal 2nd and 3rd coordinates is the given value in the radial direction. It means that the different modified structures can be connected easily if their input parameters are organized, but it also has a side effect, that the thickness of the structure is not constant in the modified structures, as seen in Figure 5.8.

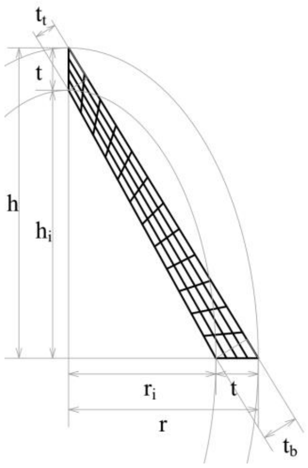


FIGURE 5.8. THICKNESS OF THE CONE AND PYRAMID TYPE DOMES

$$\frac{t_t}{t_b} = \frac{r}{h} \tag{6}$$

where

- t_t is the top thickness of the structure;
- and t_b is the bottom thickness of the structure.

The modification method works correctly if both of the second and third coordinates of the vertices are less than 180°.

5.2.2.1 TYPE 1- CUPOLA

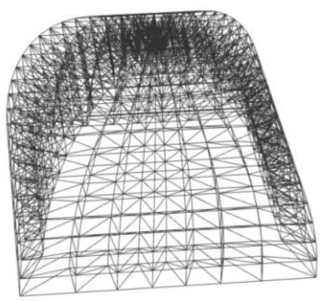


FIGURE 5.9. CUPOLA TYPE DOME

For the cupola type domes, as it can be seen in Figure 5.9. and 5. 10., the second coordinates (U_2) of the domes are changed from angular to linear coordinates. In this case the first - which were linear originally - and second coordinates (U_1, U_2) of this structure are linear, the third (U_3) is angular.

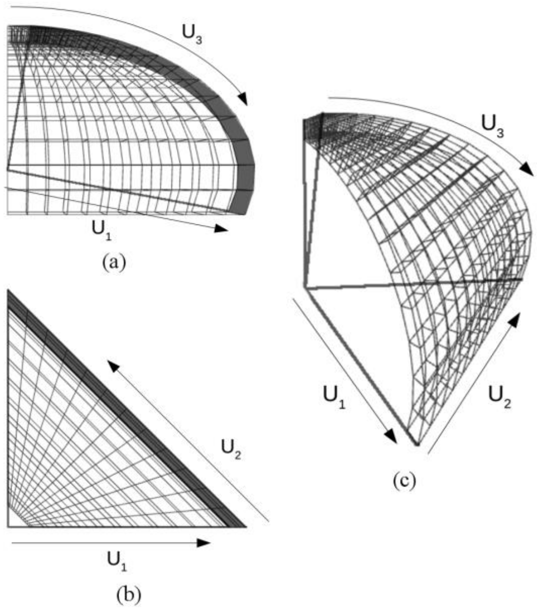


FIGURE 5.10. COORDINATES OF CUPOLA TYPE DOMES (A) FRONT VIEW, (B) TOP VIEW, (C) PERSPECTIVE VIEW

5.2.2.2 TYPE 2 - CONE

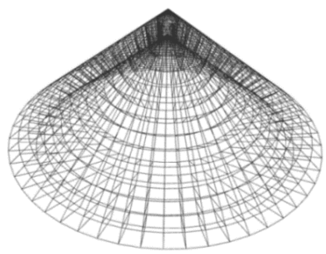


FIGURE 5.11. CONE TYPE DOME

For the cone type domes, as it can be seen in Figure 5.11. and 5. 12., the third coordinates (U_3) of the domes are changed from angular to linear coordinates. In this case the first and third coordinates (U_1, U_3) of this structure are linear, the second (U_2) is angular.

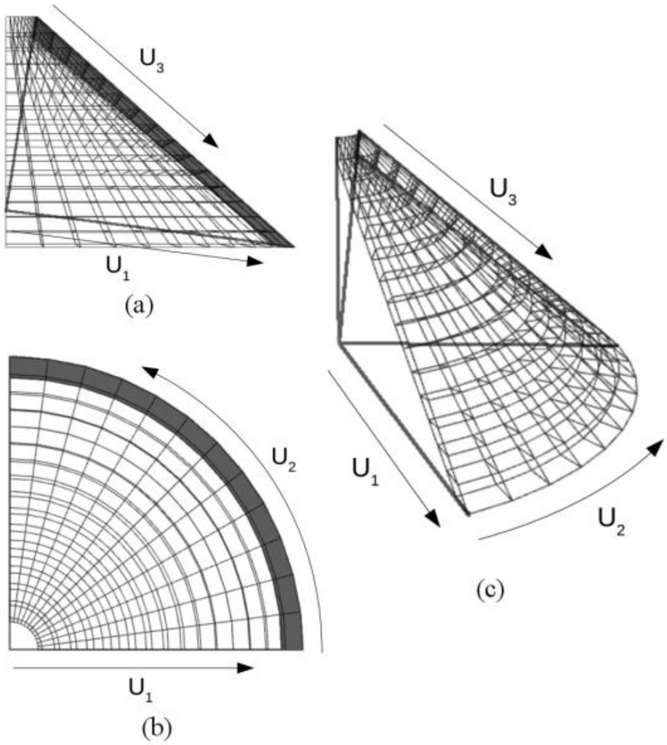


FIGURE 5.13. COORDINATES OF CONE TYPE DOMES (A) FRONT VIEW, (B) TOP VIEW, (C) PERSPECTIVE VIEW

5.2.2.3 TYPE 3 – PYRAMID

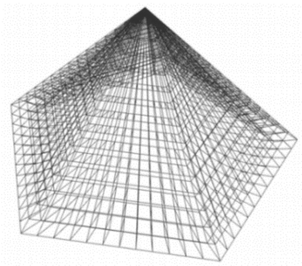


FIGURE 5.12. PYRAMID TYPE DOME

For the pyramid type domes, as it can be seen in Figure 5.13. and 5. 14., the second and third coordinates (U_2, U_3) of the domes are changed from angular to linear coordinates. In this case both three coordinates (U_1, U_2, U_3) of this structure are linear.

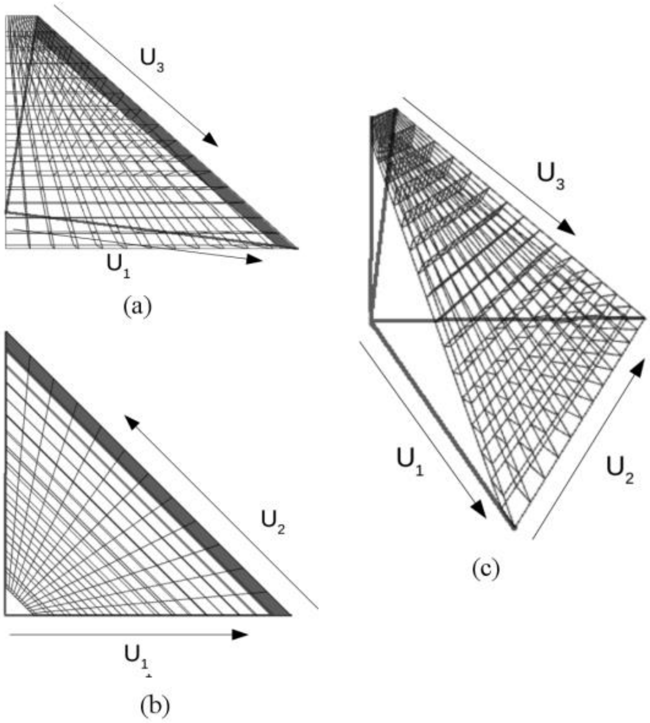


FIGURE 5.14. COORDINATES OF PYRAMID TYPE DOMES (A) FRONT VIEW, (B) TOP VIEW, (C) PERSPECTIVE VIEW

5.3 VAULT STRUCTURES

5.3.1 MATHEMATIC CALCULATIONS

The calculation of vault and polar grid structures is based on cylindrical coordinates. In Formex algebra the vault and the polar grid tools are different functions, the first of them uses polar coordinates, and the second uses cylindrical coordinates. In this adaptation they also appear as different components, although their mathematical basis is exactly the same, only the transformation of the directions is different, as it can be seen in Figure 5.15. In Formex algebra the user has to define two variables while creating polar grids, and three variables while creating vaults. In the first case the first variable is the linear scaling factor in the first (radial) direction of the polar coordinate system, the second variable is the angular scaling in the second (circumferential) direction. By the vault tool the variables are b1 (factor for scaling in the first direction (linear scale factor)), b2 (factor for scaling in the second direction (angular scale factor)), and b3 (factor for scaling in the third direction (linear scale factor))' [32]. Every other attribute of the vaults and polar grids are given by the attributes of the initial grid, for example the thickness of the double layer grids. The advantage of this solution is that it uses just a few variables, but it requires a lot calculation from the user.

To make a versatile and user-friendly tool in the calculation method some modifications can be applied and the input parameters are selected differently.

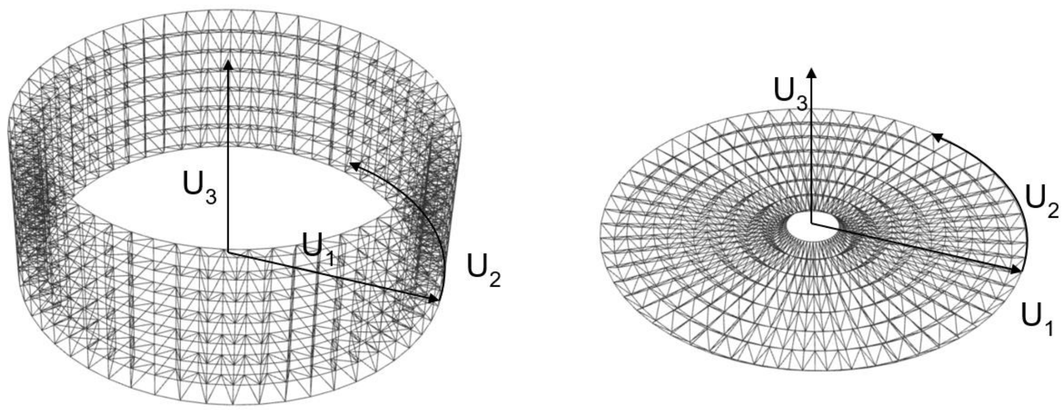


FIGURE 5.15. CYLINDRICAL COORDINATES

The coordinates of the cylindrical coordinate system are calculated from the original coordinates of the initial grid visible in Figure 5.3. The radius of the vault, and the angular and linear scaling factors, are calculated differently by the vault structures (6.6), and by the polar grids (6.7), but both of them is based on cylindrical coordinate system. The polar grid structure is created with cylindrical coordinates too, because in that way the user has easier control over the thickness of the structure,

$$U_1 = R = a * z \quad (6.6)$$
$$U_2 = \beta * x$$
$$U_3 = c * y$$

$$U_1 = R = a * y \quad (6.7)$$
$$U_2 = \beta * x$$
$$U_3 = c * z$$

where

- U_1, U_2, U_3 are the coordinates of the cylindrical coordinate system;
- *x*, *y*, *z* are the Cartesian coordinates of the initial grid;
- *R* is the radius of the structure;
- *a* and c are linear scaling factors in the first and third direction;
- and β is the factor of angular scaling in the second direction.

Instead of using a scaling factor to define coordinates (U_1, U_2, U_3), the actual radius, height, and full angular size of the structure are the variables, so the user does not have to calculate the scaling factors. Because of this calculation method the thickness of the double layer grid is also a required variable.

In Formex algebra it is possible to produce a gap in the center of the polar grid structures, which is given by the position of the initial grid according to the origin. In this adaptation a more user-friendly solution is used, there is a variable to define the size of the gap. In the case of polar grids, the size of the gap is a linear value unlike the gap of the dome structures, where it is an angular one. If a gap is used in a polar grid structure the calculation of the U_1 variable is:

$$U_{1P} = g + a * y_P \quad (6.8)$$

where

- U_{1P} is the first cylindrical coordinate of a point in the grid;
- *g* is the size of the gap;
- and y_P is the y coordinate of a point in the initial grid.

5.3.2 MODIFIED STRUCTURES

In order to create a diverse design tool some modifications of the cylindrical coordinate system can also be considered. By using these modifications, it is possible to create some additional and new structures.

This modification is based on the transformation of the angular coordinate (U_2) to linear coordinate. The corners of the initial cylindrical structure stay unchanged during the modification, and the second direction is modified from angular to linear direction by connecting the matching corners with a straight line instead of a circular curve.

The value of the linear second (U_2) coordinate of the modified structure is based on the ratio of the initial angular coordinates of the direction. The linear coordinate divides the linear edge of the initial grid in the same ratio as the initial angular coordinate of the vertex divided the interval of the angular coordinates of the initial grid, as it can be seen in Figure 5.16.

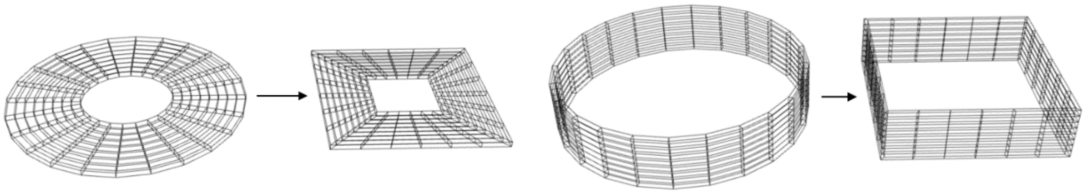


FIGURE 5.16. MODIFIED VAULT STRUCTURES

This modification method has a consequence that that the resulting structures can be combined and joined in an easier way. If the value of the radius, thickness and gap size are synchronized, the structures can be joined precisely.

At the same time this modification has a result that the thickness of the modified structures differs from the variable 'thickness'.

5.3.3 COMBINED STRUCTURES

With these tools it is very easy to create combined structures. Some examples are shown in Figure 5.17.

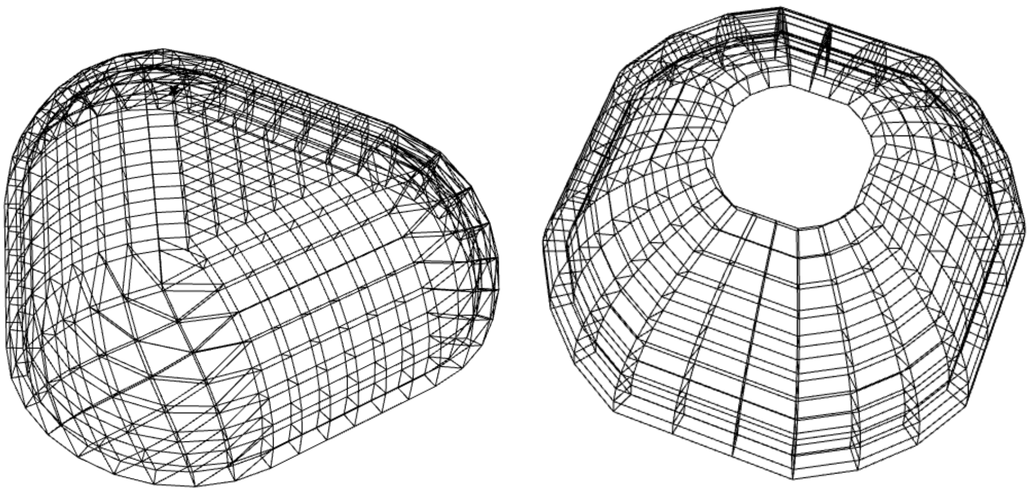


FIGURE 5.17. COMBINED STRUCTURES.
ON LEFT: TRIANGULAR DOME + VAULT + PLANAR GRID. ON RIGHT: DOME + CUPOLA

5.4 ROTATIONAL GRIDS

5.4.1 MATHEMATICAL CALCULATIONS

Based on the vault tool two types of rotational grid tools are also created. These components transform a grid given in Cartesian coordinate system to cover a surface of revolution. The generating curve [17] of the surface has to be a planar curve. The user can draw it in the *xy* plane, and the axis *y* represents the axis of the surface of revolution.

The ratio of the *y* coordinate of the point in the initial grid compared to the minimum and maximum *y* value shows the position of the point on the generating curve:

$$e = \frac{y}{y_{max} - y_{min}} \quad (6.9)$$

where

- e is the value of the ratio along the generating curve;
- *y* is the original y coordinate of the point;
- and y_{min} and y_{max} are the smallest and biggest y coordinates in the initial grid.

This ratio is a number between 0 and 1, and if it is 0, the point is on the start of the curve, if it is 1, then it is at the end of the curve. The ratio changes between the two extreme values and it divides the curve in that ratio, based on the length of the curve. The distance of this point from the axis y is measured (x_{cp}).

The gap in this shape is not a variable; it is calculated based on the distance of the generating curve from axis *y* as it is plotted in Figure 5.18.,

$$g = x_{c2} * \frac{r}{x_{c1}} \quad (6.10)$$

where

- x_{c1} and x_{c2} are the smallest and the biggest x coordinates of the generating curve;
- *r* is the biggest radius of the final structure given by a variable;
- and *g* is the gap of the final structure.

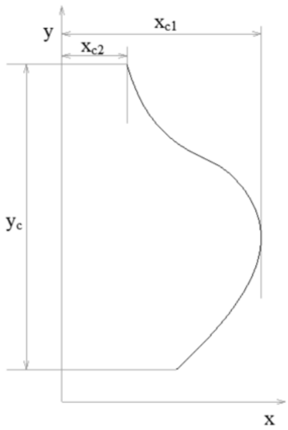


FIGURE 5.18. GENERATRIX CURVE

The larger radius and the height of the structure are input parameters, and the U_1 coordinates of the points are calculated as:

$$U_{1P} = \frac{(x_{cp}-x_{c2})(r-g)}{x_{c1}-x_{c2}} \quad (6.11)$$

where

- U_{1P} is the first cylindrical coordinate of a point;
- and x_{cp} is the x coordinate of the point projected to the generating curve.

The U_2 coordinates are calculated in the same way as introduced by the vault tool introduced in (6.6). The two components differ in the calculation of the third coordinate (U_3) and the thickness of the structure.

Both of these tools also contain a modified version, where the top view of the structure is polygonal.

5.4.2 FIRST ROTATIONAL GRID TOOL

In the first rotational grid tool which is visible in Figure 5.19., the thickness is horizontal and the horizontal distance between the sections is equal, if the distance of these sections was equal in the initial grid.

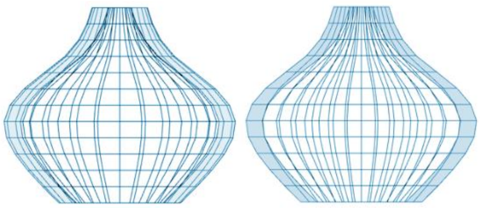


FIGURE 5.19. SIDE VIEW OF GRID CREATED WITH THE 1ST GRID OF REVOLUTION TOOL

The ratio of the third coordinate (U_3) of a point in the z direction is the same as it was to the y direction of the initial grid. It is calculated as:

$$U_3 = U_{3max} * e \quad (6.12)$$

Following this logic, the thickness of the structure is the thickness in the first direction (U_1). This tool works like a cylinder where the radius varies based on the generating curve.

This tool can be useful for many architectural purposes, because in this way the grid can follow the division of the storeys as it is shown in Figure 5.20. This tool has a modified variant too which is visible in Figure 5.21.

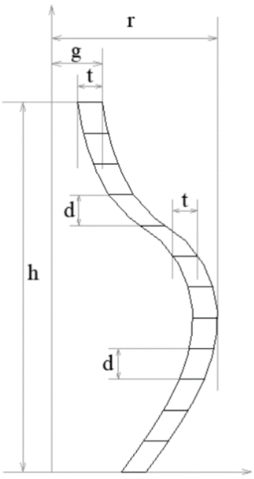


FIGURE 5.20. STRUCTURE OF THE 1ST GRID OF REVOLUTION TOOL

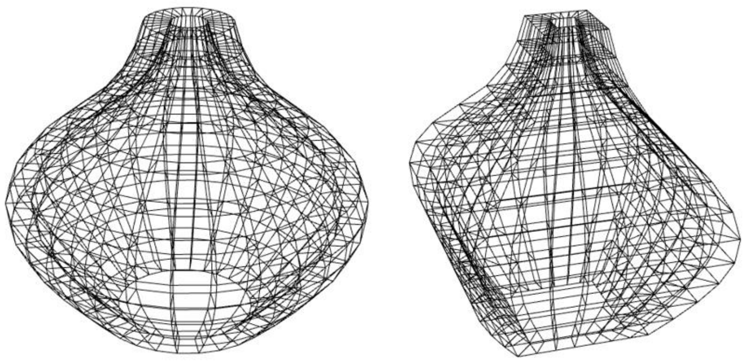


FIGURE 5.21. ORIGINAL AND MODIFIED VERSION OF A GRID OF RVOLUTION CREATED BY THE FIRST TOOL

5.4.3 SECOND ROTATIONAL GRID TOOL

The second rotational grid tool which is visible in Figure 5.22., differs more from the vault tool as the previous one. The final grid reflects much more the generating curve like it did in the previous tool.

The U_3 value is calculated based on the y coordinate of the point projected to the generating curve as:

$$U_{3P} = \frac{y_{cp}}{y_c} * U_{3max} \qquad (6.13)$$

where

- y_{cp} is the y coordinate of the point projected to the generating curve;
- and y_c is the highest value of y coordinate of the generating curve.

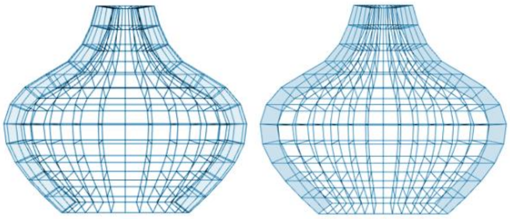


FIGURE 5.22. SIDE VIEW OF GRID CREATED WITH THE 2ND GRID OF REVOLUTION TOOL

This calculation method has a consequence that the length of the edges, which have the same length in the initial grid, will be close to each other. It is not exactly the same, because the x/y ratio of the generating curve and the height/radius ratio of the final grid can be different. At the same time if these ratios are the same, the length of these segments will be the same. It is easy to reach this state if the user draws the generating curve in the same size (or same ratio) as the required size of the final grid.

The direction of the thickness in this tool is always perpendicular to the generating curve in the given point as it can be seen in Figure 5.23. It means that it is not calculated based on cylindrical coordinates. It is created with the moving of the point into this perpendicular direction.

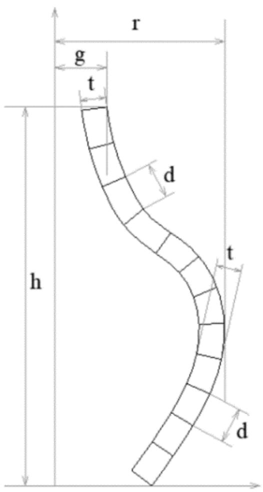


FIGURE 5.23. STRUCTURE OF THE 2ND GRID OF REVOLUTION TOOL

This solution is most useful if the equal vertical distance is not required but the uniform thickness is. This tool has a modified variant too which is visible in Figure 5.24.

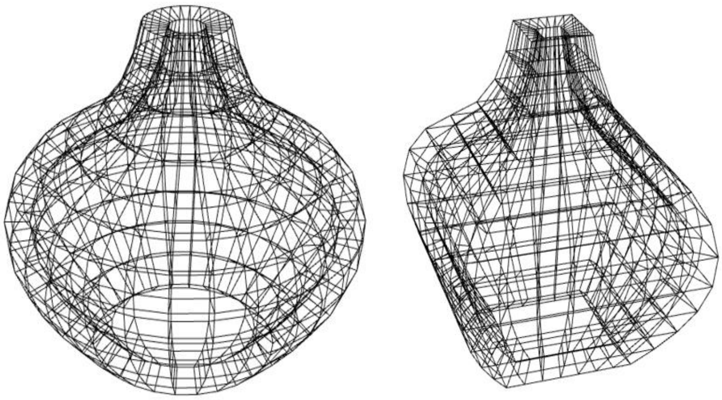


FIGURE 5.24. ORIGINAL AND MODIFIED VERSION OF A GRID OF RVOLUTION CREATED BY THE SECOND TOOL

5.4.4 ROTATIONAL GRID 1 VS GRID 2

The difference between the two rotational grid tool is clearly visible, if the end of the generating curve 'bends back' as it can be seen in Figure 5.25. The first tool does not follow this bend, as it can be seen in Figure 5.26., because the vertical position of the points always rises as the ratio of the position of a point projected to the generating curve changes. The second tool on the other hand can follow this bend as it can be seen in Figure 5.27.

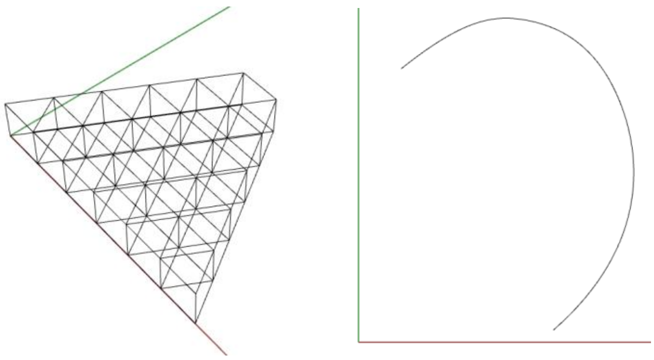


FIGURE 5.25. INITIAL GRID AND THE GENERATRIX CURVE

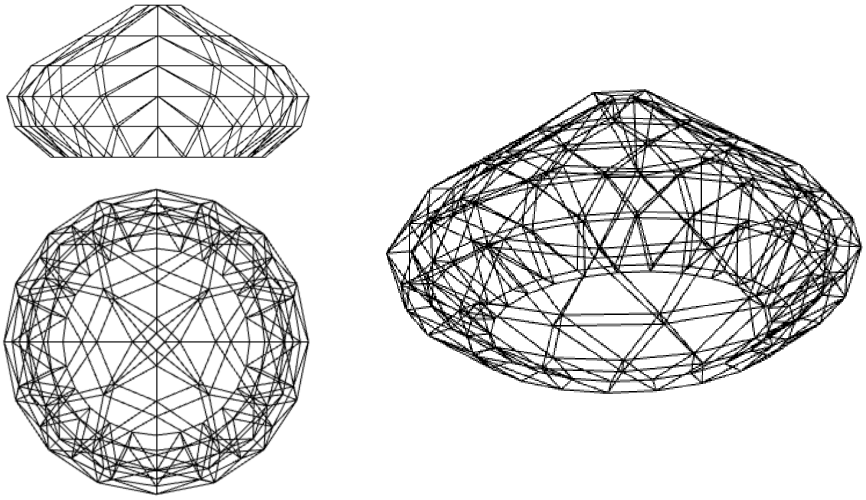


FIGURE 5.26. FRONT VIEW, TOP VIEW AND PERSPECTIVE VIEW OF THE GRID OF REVOLUTION CREATED WITH THE FIRST TOOL

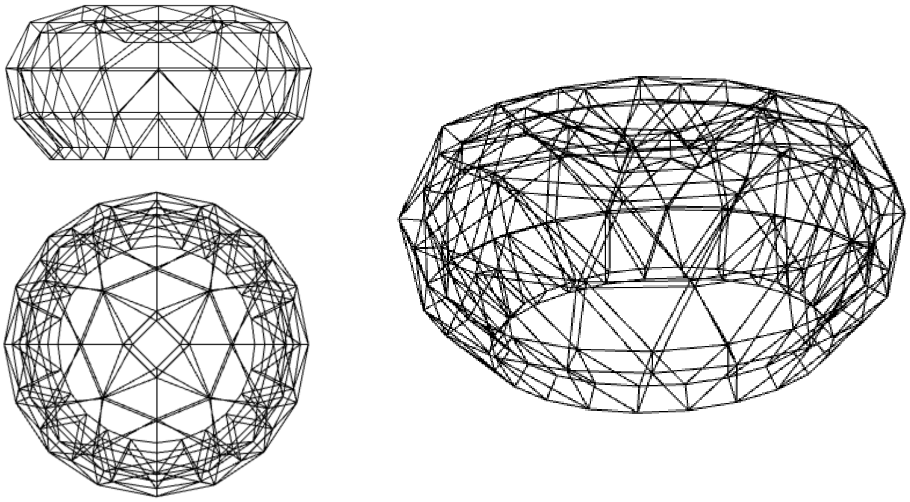


FIGURE 5.27. FRONT VIEW, TOP VIEW AND PERSPECTIVE VIEW OF THE GRID OF REVOLUTION CREATED WITH THE FIRST TOOL

5.5 TRIANGULAR GRIDS

Based on the concept of Diamatic domes a triangular version of every component can also be created. Instead of changing of the calculation of the coordinate system transformations themselves an additional coordinate system transformation was added which transforms the skew coordinates of a triangular grid to Cartesian coordinates. After this transformation the same calculation method can be used to create dome, vault, polar grid or rotational grid structures as, it can be seen in Figure 5.28.

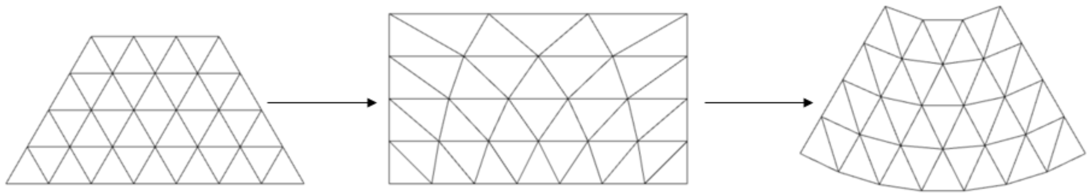


FIGURE 5.28. TRANSFORMATION OF A TRIANGULAR GRID TO A CARTESIAN THAN TO A POLAR GRID

To create a Cartesian grid from a skew grid the triangular grid had to be a part of an isosceles triangle. The base of the triangle is chosen to be parallel with axis x, as it can be seen in Figure 5.29.

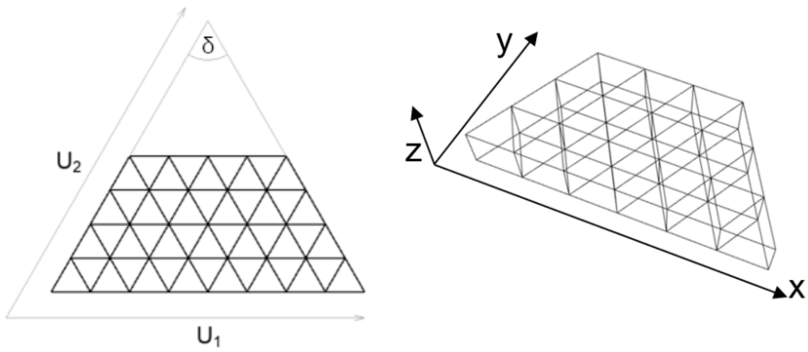


FIGURE 5.29. INITIAL TRIANGULAR GRID

First the skew coordinates of the triangular grid based on its Cartesian coordinates can be calculated as:

$$U_1 = x - \frac{y}{\tan\delta} \qquad (6.14)$$
$$U_2 = \frac{y}{\sin\delta}$$
$$U_3 = z$$

where

- U_1, U_2, U_3 are the skew coordinates of the grid;

- x, y, z are the coordinates of the original grid in Cartesian coordinate system;
- and δ is the angle of the apex of the isosceles triangle.

In the next step the triangular grid is transformed into a rectangular shape based on its skew coordinates as:

$$x_c = \frac{U_1}{U_{1r}} U_{1max} \quad (6.15)$$
$$y_c = U_2$$
$$z_c = U_3$$

where

- x_c, y_c, z_c are the Cartesian coordinates of the transformed grid;
- U_{1max} is the largest U_1 coordinate in the grid;
- and U_{1r} is the largest U_1 coordinate of the points with the same U_2 coordinate.

After this transformation these grids can be transformed with any of the developed grid tools, like the rectangular grids. In Figure 5.30. all of the triangular grid versions of the dome, vault and polar grid tools are visible.

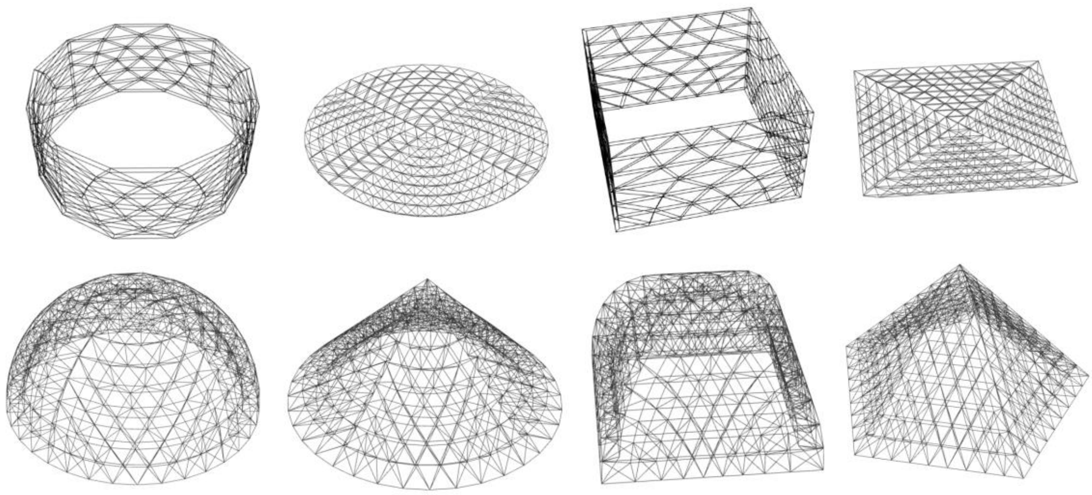


FIGURE 5.30. TRIANGULAR GRID VERSION OF THE VAULT, POLAR GRID AND DOME TOOLS

This version of the rotational grid tools was also created as it can be seen in Figure 5.31.

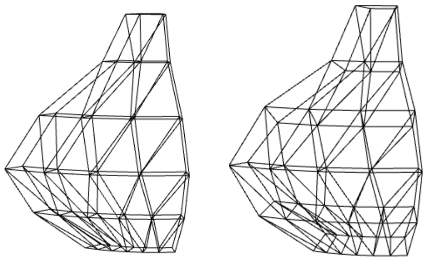


FIGURE 5.31. TRIANGULAR GRID VERSION OF THE GRID OF REVOLUTION TOOLS

5.6 ABOUT GRASSHOPPER

The previously described shape creating calculations were implemented in grasshopper. Grasshopper is a graphical algorithm editor which works together with the 3D modelling program Rhino 3D [15]. It uses the syntax of Rhino. With grasshopper it is possible to give algorithm controlled commands to Rhino. Because of its graphical interface no programing language is necessary, although users have to understand and use the workflow of programming. Grasshopper contains a lot of components with which users can create special forms and fluid geometries. Grasshopper can also communicate with a lot of other design software. Because of this grasshopper is an ideal environment for the coordinate system transformations known from Formex algebra. Designers, who usually do not have programming skills can use this environment easier, and with the various geometrical components of grasshopper the efficiency of this tool becomes apparent. Because the adaptation was created to keep the original data structure of the initial grid the users can manipulate it easily and create structures plotted in Figure 5.32.

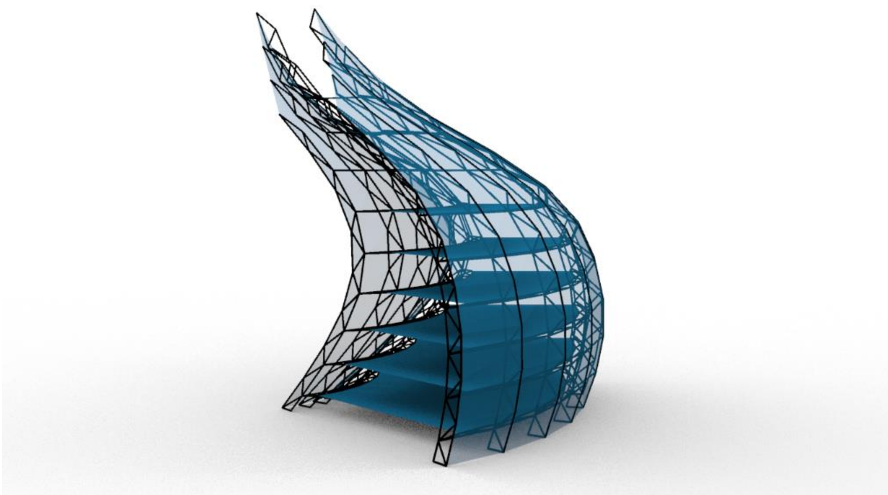


FIGURE 5.32. SIMPLIFIED BUILDING MODEL

5.6.1 USER INTERFACE

This set of coordinate system transformation tools contains 10 transformation components and one settings component as seen in Figure 5.33. The transformation components are the Dome, the Vault, the Polar Grid tools, the two versions of the rotational grid tool and the triangular version of all of these. The input parameters of these tools are the initial grid and the basic variables of the structure. These are the necessary settings for this tool. The settings tool can be used for every transformation tool to add extra setting options as it can be seen in Figure 5.34. The gap variable of it is special, because it is angular value in the dome tool, linear value in the polar grid tool and the other tools disregard this variable.

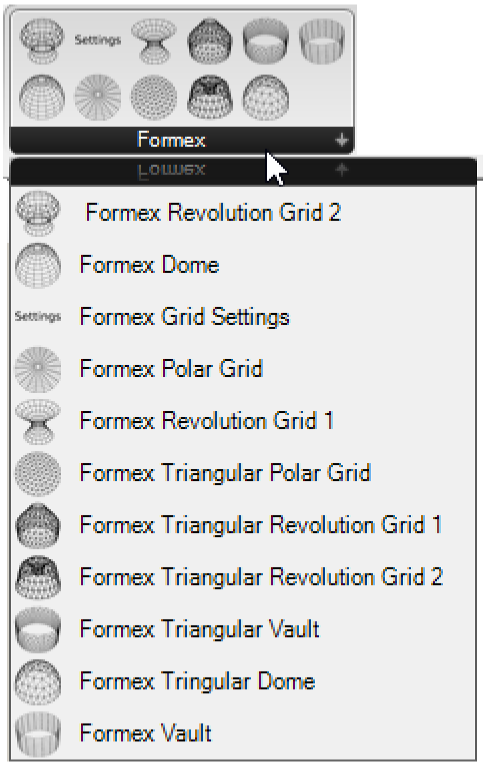


FIGURE 5.33. COORDINATE SYSTEM TRANSFORMATION TOOLS ADAPTED INTO GRASSHOPPER

The users can use the already existing components of grasshopper to create the initial grid in Cartesian coordinate system. The grids can be modified further with the grasshopper components. The transformation components are created to keep the data structure of the initial grid during the transformation. This is very useful, because the user can group the lines of the initial grid as it is necessary after the transformation.

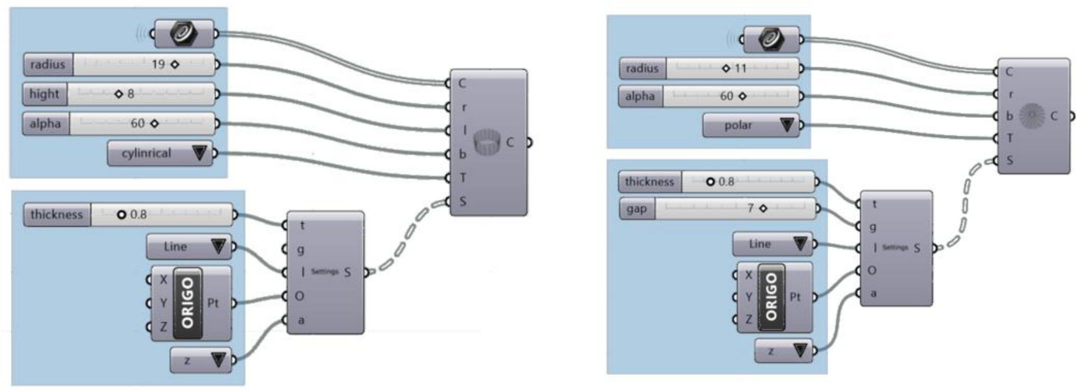


FIGURE 5.34. GRASSHOPPER USER INTERFACE

6 CORRELATION BETWEEN THE STATICAL BEHAVIOUR OF DOME STRUCTURES AND THEIR GRAPH CHARACTERISTICS

Graph theory is a branch of mathematics which is useful in many fields of science. Graph characteristics can tell a lot about the behavior of complex systems. The graph representation of dome and grid structures can be used for different purposes, like the examination of the rigidity of structures, flexibility and dynamic analysis, optimization, and many other fields [39, 40]. In this research three sets of dome structures and their graph characteristics were analyzed. The first set of domes contains 13 different single-layer domes with hinged joints, the second set contains the same domes with fix joints, and the third set of domes contains 16 double-layer domes with hinged joints.

6.1 ANALYSIS OF DOME STRUCTURES

For the modelling and structural analysis of the dome structures a NURBS-based computer graphic software Rhinoceros 3D [15] and its plug-ins were used. The models were primarily made with the plug-in Grasshopper [16], which is a graphical algorithm editor, and the components which were introduced in the previous chapter based on Formex algebra [32, 33].

6.2 STRUCTURAL ANALYSIS

The structural analysis of the dome structures is calculated with the finite element modelling software AxisVM [41]. For the structural analysis uniform parameters were used for every dome structure. All the beams have the same circular cross section (diameter = 5 cm, thickness = 1 cm), and the same material (steel S235). All the joints are hinged joints in the first and third set, which can transfer both axial and shear forces, but cannot transfer either torsional or bending moments. The joints are fix joints in the second set which can transfer both axial and shear forces, and torsional and bending moments too. Gravity load and snow load is simulated, the characteristic value of the latter is 1.25 kN/m^2 in Hungary. It is calculated as a surface load transformed into equivalent element loads by the algorithm. The domes form a half sphere, the radius and height is 10 meter. The thickness of the double layer grids is 1 meter. All of the bottom nodes are supported, which are fix to translations into the x, y and z directions, but free for rotation into any direction. The analyzing algorithm uses second order theory to calculate deflections. To be able to compare the different structures, the single layer domes were designed to weigh more than 10 000, but less than 12 000 kilograms. The weight of the double layer domes is between 20 000 and 23 000 kg. Because of this the differences in the statical behavior cannot be explained only with the thicker structure or the smaller weight load.

The analyzed value for ranking structures are the maximum displacement (d), which is calculated in the endpoints and the midpoints of the elements. This value was calculated both with unilateral and full surface snow load.

6.3 GRAPH REPRESENTATION OF DOMES AND ANALYSED CHARACTERISTICS

To be able to analyze the graph characteristics, the domes are interpreted as undirected simple planar graphs. This means that edges have no orientation, both multiple edges and loops are disallowed and vertices and edges can be drawn in a plane as long as no edge intersects with any other edge.

For the research the number of the vertices and edges, the average degree and the average clustering coefficient is calculated. The degree of a vertex is the number of edges incident to the vertex [42]. The average degree is defined as:

$$\langle k \rangle = \frac{1}{N}\sum_{i=1}^{N} k_i = \frac{2L}{N} \quad (1)$$

- k_i is the number of the neighbors of v_i,
- L is the number of the edges and
- N is the number of the vertices in the graph.

The clustering coefficient shows how densely the neighbors of a vertex connect to each other [25]. The clustering coefficient of a node is defined as:

$$C_i = \frac{2L_i}{k_i(k_i - 1)} \quad (2)$$

- k_i is the number of the neighbors of v_i, and
- L_i is the number of the connections between these neighbors.

The average clustering coefficient is the average of the local clustering coefficient of the nodes:

$$\langle C \rangle = \frac{1}{N}\sum_{i=1}^{N} C_i \quad (3)$$

Since the double-layer domes are space structures, their graphs are not planar graphs. As it was discussed in chapter 4, besides vertices, edges and faces space structures have cells too. For this research the cells of the double-layer domes, and their graph characteristics were also analyzed. The cells of these domes are visible in Figure 6.1.

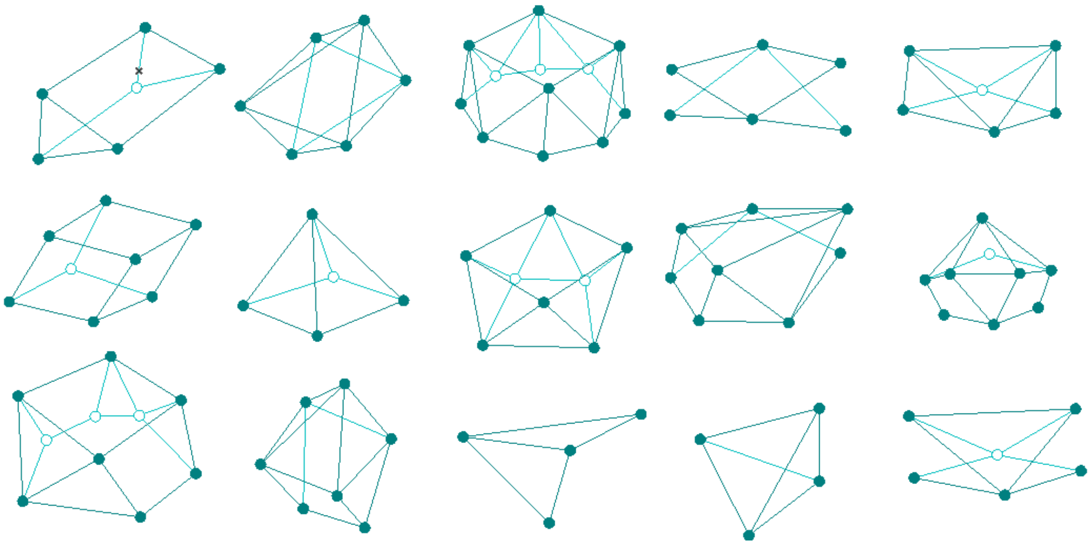


FIGURE 6.1 CELLS OF DOUBLE LAYER DOMES

6.4 ANALYSED DOME STRUCTURES

The analyzed domes are versions of the most used dome types, Schwedler, ribbed, lamella and Diamatic domes based on the dome types shown in [32, 33]. Dome 1-4 are different Schwedler domes, dome 5 and 6 are ribbed domes, dome 7-10 are some variations of lamella domes. Dome 11 is a Diamatic dome, Dome 12 and 13 are also based on Diamatic dome structures, as it can be seen in Figure 6.2.

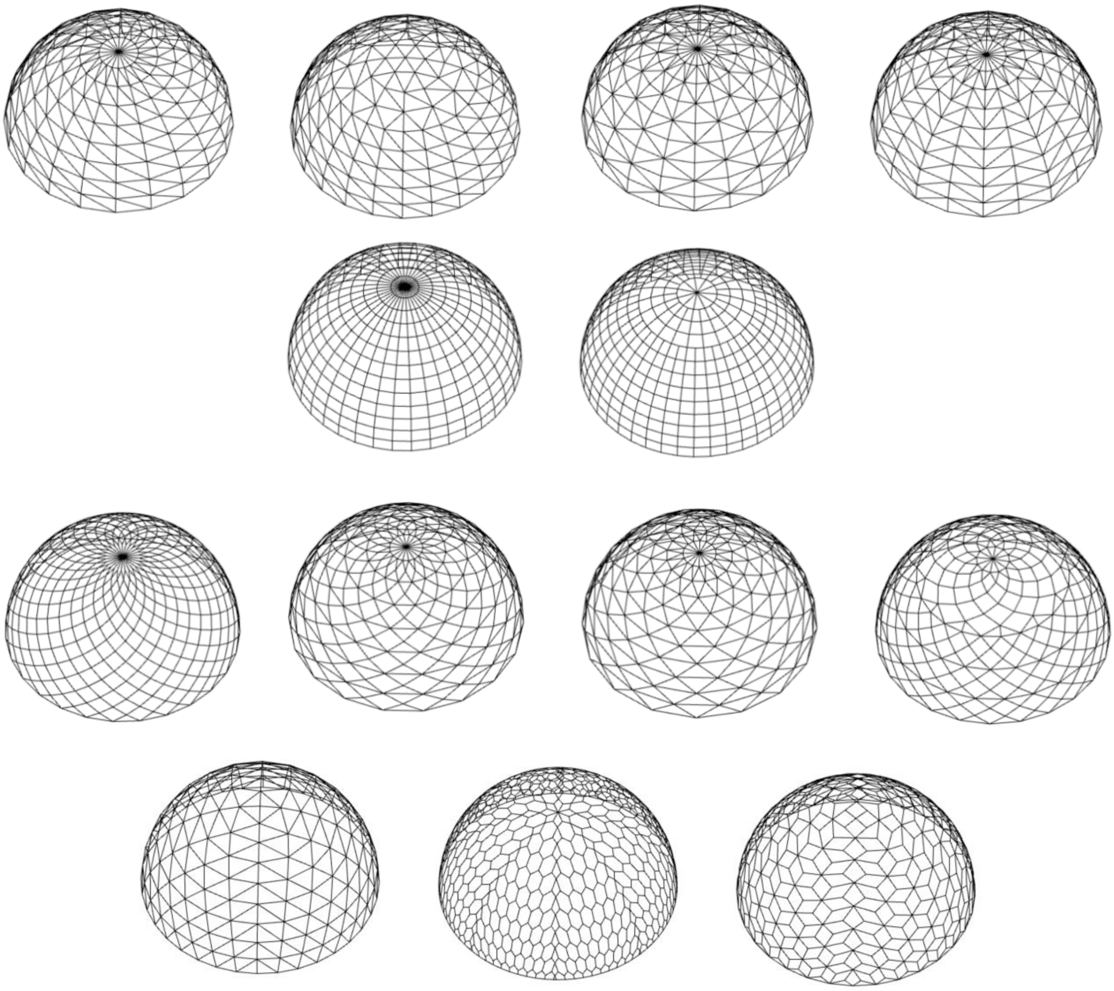


FIGURE 6.2. ANALYZED SINGLE-LAYER DOMES.
1ST ROW: 1-4, 2ND ROW: 5-6, 3RD ROW: 7-10, 4TH ROW: 11-13

The analyzed double-layer grids are the modified versions of the single-layer domes, as it can be seen in Figure 6.3. 8A and 8B are very similar, the only difference is, that 8B contains more beams in the intermediate layer between the outside and inside elements. 9A and 9B are also only different in this matter.

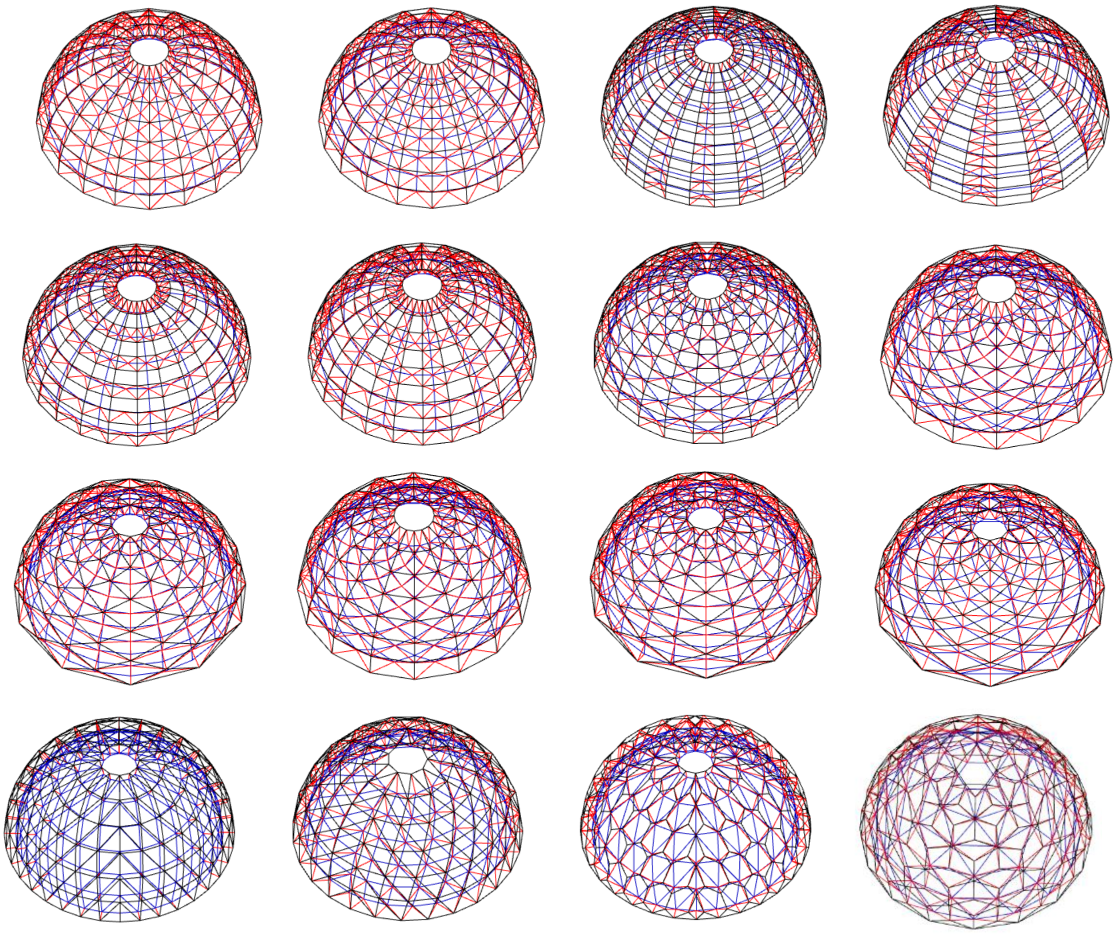


FIGURE 6.3. ANALYZED DOUBLE-LAYER DOMES.
1ST ROW: 1-4, 2ND ROW: 5-7, 8A, 3RD ROW: 9A, 8B, 9B, 10 4TH ROW: 11-14

6.5 RESULTS

For the single-layer domes the average degree and the average clustering coefficient of the structure were calculated, as it can be seen in Table 6.1. Table 6.2. shows the results of the double-layer grids. In this case not just the average, but also the minimum value of the degree and the clustering coefficient were calculated. The graph characteristics of the cells of the double-layer grids can be seen in Table 6.3.

Dome ID	Graph characteristics				m (kg) mass	HINGED joints			FIX joints		
	N number of vertices	L number of edges	<k> average degree	<C> average clustering coefficient		d (mm) maximum displacement unilateral snow load	d (mm) maximum displacement full surface snow load	d (mm) average displacement	d (mm) maximum displacement unilateral snow load	d (mm) maximum displacement full surface snow load	d (mm) average displacement
1	181	520	5,75	0,42	11474,62	0,61	1,77	1,19	1,93	4,22	3,08
2	175	498	5,69	0,42	11095,91	1,61	1,65	1,63	6,72	5,26	5,99
3	181	520	5,75	0,49	11474,62	1,62	1,72	1,67	7,28	4,01	5,65
4	181	520	5,75	0,43	11474,62	0,39	1,79	1,09	0,91	4,09	2,50
5	495	988	3,99	0,03	11394,39	instability	instability	instability	16,08	1,13	8,61
6	507	979	3,86	0,00	11182,38	instability	instability	instability	19,61	2,40	11,01
7	460	918	3,99	0,03	11500,49	instability	instability	instability	291,93	560,16	426,05
8	271	666	4,92	0,31	11638,84	instability	instability	instability	36,25	57,20	46,73
9	177	512	5,79	0,42	11204,16	1,81	4,93	3,37	2,50	7,11	4,81
10	313	708	4,52	0,20	10998,08	instability	instability	instability	9,32	104,56	56,94
11	181	504	5,57	0,43	10719,81	2,12	2,14	2,13	12,21	5,75	8,98
12	969	1496	3,09	0,01	11780,76	instability	instability	instability	12,54	65,15	38,85
13	401	800	3,99	0,04	11324,72	instability	instability	instability	29,45	318,38	173,92

TABLE 6.1. GRAPH CHARACTERISTICS OF SINGLE-LAYER DOME STRUCTURES AND THE RESULTS OF THE STATICAL ANALYSIS

Dome ID	Graph characteristics						m (kg) mass	displacement		
	N number of vertices	L number of edges	k_{min} minimum degree	<k> average degree	C_{min} minimum clustering coefficient	<C> average clustering coefficient		d (mm) maximum displacement unilateral snow load	d (mm) maximum displacement full surface snow load	d (mm) average displacement
1	340	1160	5	6,82	0,36	0,45	20204,67	1,02	1,03	1,03
2	300	1120	5	7,47	0,43	0,44	20771,83	1,00	0,97	0,99
3	560	1526	4	5,45	0,14	0,20	21463,20	instability	instability	instability
4	410	1250	4	6,10	0,29	0,35	20657,63	instability	instability	instability
5	432	1332	5	6,17	0,28	0,38	20580,06	180,85	instability	instability
6	408	1344	5	6,59	0,38	0,45	21094,46	1,46	1,17	1,31
7	420	1344	4	6,40	0,29	0,32	21769,46	9,73	instability	instability
8a	336	1104	4	6,57	0,00	0,22	20831,86	1,77	2,69	2,23
8b	336	1200	5	7,14	0,29	0,38	21775,86	1,18	2,67	1,92
9a	330	1060	3	6,42	0,00	0,15	21044,47	instability	instability	instability
9b	330	1170	4	7,09	0,29	0,36	22129,57	1,80	2,43	2,12
10	220	880	7	8,00	0,40	0,46	20143,52	2,12	3,70	2,91
11	336	1080	4	6,43	0,26	0,29	21187,11	instability	instability	instability
12	243	990	5	8,15	0,40	0,45	21514,24	0,57	0,91	0,74
13	384	1160	3	6,04	0,00	0,25	21273,16	instability	instability	instability
14	242	858	4	7,09	0,00	0,20	20793,05	2,10	instability	instability

TABLE 6.2. GRAPH CHARACTERISTICS OF DOUBLE-LAYER DOME STRUCTURES AND THE RESULTS OF THE STATICAL ANALYSIS

Dome ID	Graph characteristics of cells						m (kg) mass	displacement		
	<F> average number of faces	<f> average number of vertices of a face	<k> average degree	k_{min} minimum degree	<C> average clustering coefficient	C_{min} minimum clustering coefficient		d (mm) maximum displacement unilateral snow load	d (mm) maximum displacement full surface snow load	d (mm) average displacement
1	4,09	3,16	2,94	2,50	0,84	0,67	20204,67	1,02	1,03	1,03
2	4,35	3,07	3,07	3,00	0,88	0,67	20771,83	1,00	0,97	0,99
3	5,23	3,59	3,05	3,00	0,34	0,00	21463,20	instability	instability	instability
4	4,78	3,30	3,07	3,00	0,64	0,00	20657,63	instability	instability	instability
5	5,38	3,25	3,16	3,00	0,69	0,33	20580,06	180,85	instability	instability
6	5,51	3,12	3,12	3,00	0,82	0,52	21094,46	1,46	1,17	1,31
7	7,32	3,20	3,57	3,20	0,58	0,47	21769,46	9,73	instability	instability
8a	4,71	3,39	3,01	2,50	0,52	0,27	20831,86	1,77	2,69	2,23
8b	6,00	3,15	3,44	3,00	0,67	0,61	21775,86	1,18	2,67	1,92
9a	4,94	3,45	3,06	3,00	0,43	0,27	21044,47	instability	instability	instability
9b	6,23	3,15	3,49	3,00	0,65	0,61	22129,57	1,80	2,43	2,12
10	4,43	3,09	3,09	3,00	0,86	0,67	20143,52	2,12	3,70	2,91
11	5,00	3,60	3,00	3,00	0,33	0,33	21187,11	instability	instability	instability
12	5,56	3,01	3,39	3,00	0,86	0,67	21514,24	0,57	0,91	0,74
13	5,62	3,35	3,14	2,67	0,61	0,00	21273,16	instability	instability	instability
14	4,58	3,35	3,00	3,00	0,57	0,27	20793,05	2,10	instability	instability

TABLE 6.3. GRAPH CHARACTERISTICS OF CELLS OF DOUBLE-LAYER DOME STRUCTURES AND THE RESULTS OF THE STATICAL ANALYSIS

6.6 CORRELATION

Single-layer truss-grid domes (with hinges) show clear correlation between the average degree and average displacement, as it can be seen in Figure 6.4. If the average degree is lower than 5, the domes are instable, if it is bigger than 5,5, they are maximum displacement is below 5 cm. If the joints are fix, the statical behavior of some of these domes changes. Although the maximum displacement of dome 5 and 6 is above 10 cm, it is still below 20 cm, so they can be also considered as stable. Some of the data suggests exponential correlation between the average degree and the average displacement, but domes 5, 6 and 11 are far from the exponential curve.

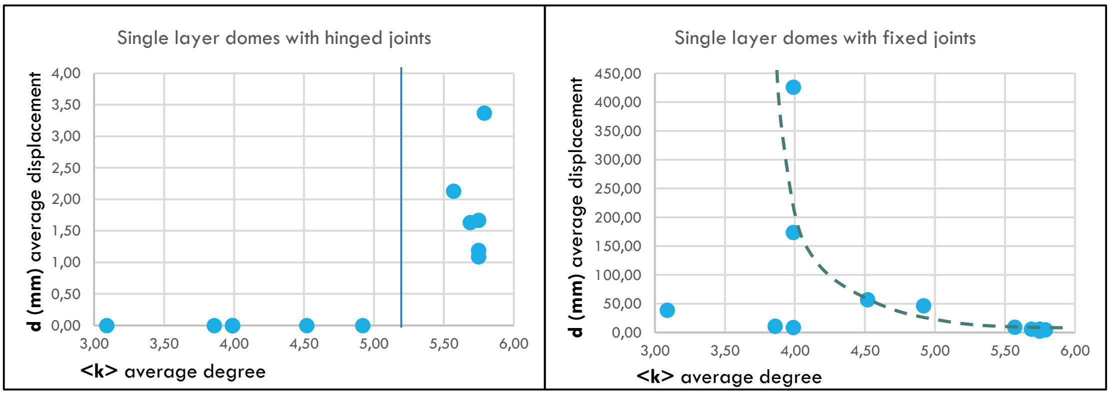


FIGURE 6.4. AVERAGE DISPLACEMENT WITH AVERAGE DEGREE OF SINGLE-LAYER DOMES

The correlation between the average clustering coefficient and average displacement is very similar to the previous data, as it can be seen in Figure 6.5. If the joints are hinged, the correlation is clear. The domes with a clustering coefficient lower than 0,35 are instable. If the average clustering coefficient is bigger than 0,4, the maximum displacement is smaller than 5 cm. If the joints are fix some data suggests exponential connection again, but domes 5, 6 and 11 are exceptions again. If a dome was stable with hinged joints, it can be considered stable with fix joints too, but the maximum displacement is usually bigger.

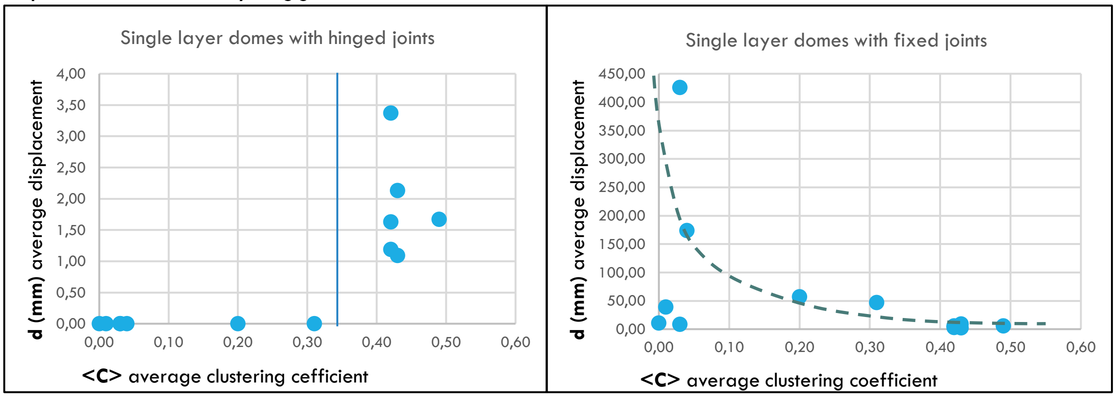


FIGURE 6.5. AVERAGE DISPLACEMENT WITH AVERAGE CLUSTERING COEFFICIENT OF SINGLE-LAYER DOMES

Double-layer grids show some correlation between some graph characteristics and the average displacement of the structures, but on or two of the studied structures always act like exceptions. If the average degree is lower than 6,5, the structures are instable, if it is bigger, they are usually stable. At the same time the average degree of dome 14 is above 7, it is still instable, as it can be seen in Figure 6.6. The domes with minimum clustering coefficient 0,29 can be either stable or instable, below it they are usually instable, above it they are stable. In this case the exceptional data is the dome 8a, its minimum clustering coefficient is 0, still the maximum displacement is below 3 cm, as it can be seen in Figure 6.6

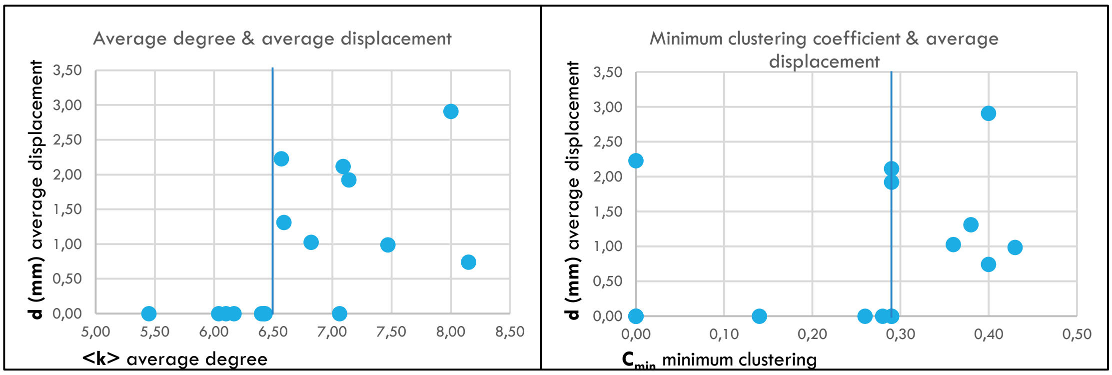


FIGURE 6.6. AVERAGE DISPLACEMENT WITH AVERAGE DEGREE AND MINIMUM CLUSTERING COEFFICIENT OF DOUBLE-LAYER DOMES

Figure 6.7. shows the average displacement compared to the graph characteristics of the cells. These show very similar correlation to the previous one, the exception is in both cases 8a, which average number of a face in a cell is above 3,2, its minimum clustering coefficient of cells is below 0,5, it is still a stable structure.

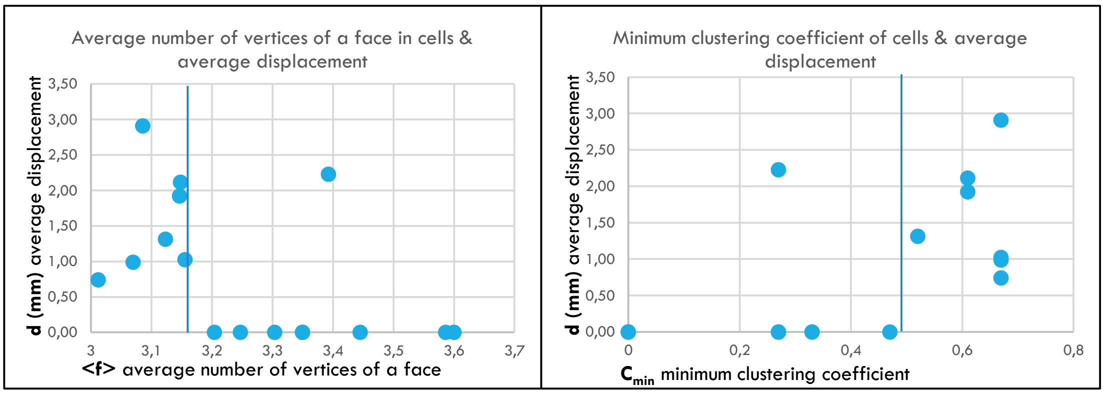


FIGURE 6.7. AVERAGE DISPLACEMENT WITH AVERAGE NUMBER OF VERTICES OF A FACE IN CELLS AND MINIMUM CLUSTERING COEFFICIENTOF A CELL OF DOUBLE-LAYER DOMES

6.7 CONCLUSIONS

This research shows clear correlation between the average degree and the average clustering coefficient of the graph representation of single-layer truss-grid dome structures and their maximum displacement under snow load. It also shows, that if a single layer dome is stable with hinged joints, it will remain stable with fix joints too, but the value of the displacement is usually bigger. At the same time some domes, which were instable with hinged joints become stable with fix joints, but not all of them. Single layer domes with fix joints not show clear correlation between their graph characteristics and displacement.

Double-layer truss-grid domes do not show unexceptional correlation with their graph characteristics. The reason is, that these values can predict if a structure is generically rigid [40], but they cannot indicate it clearly. In Figure 6.8. all of the planar grids are generically rigid, still, the clustering coefficient of the dark point varies from 1 to 0, which are the possible extreme values of this variable.

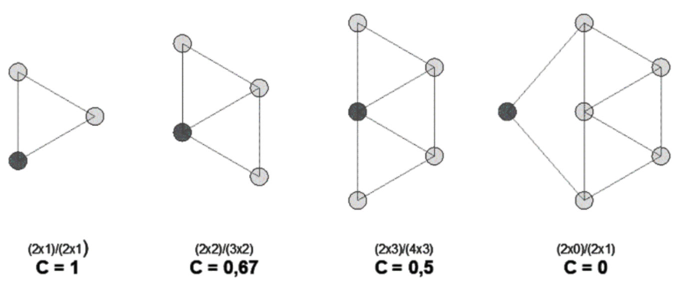


FIGURE 6.8. CLUSTERING COEFFICIENT OF THE DARK POINT

Although double-layer truss-grid domes the correlation is not without exception, it still shows probability, so these values can be used to guide the design of these dome structures.

7 TEACHING PARAMETRIC DESIGN

7.1 INTRODUCTION

Parametric design is a new field of architecture, it requires new capabilities from architects, and teaching parametric design constitutes new challenges for teachers.

The subject which covers parametric architectural design was called at the University of Pécs "Parametric constructional design", "Parametric design" or "Constructional skills (parametric design)" in the different semesters. It is obligatory for students on Architecture MSc and Architectural Art MA majors and optional for bachelor students. Master students learn parametric design in the first semester of their master studies. The applied software is Rhino 3D [15] and Grasshopper [16], which is a plugin for Rhino.

7.2 FIELDS OF KNOWLEDGE

The knowledge of this subject mostly covers fields which are specific for parametric architecture. At the same time, it relies on skills which students learned earlier, as a part of their traditional base subjects. From traditional architectural skills the geometrical and spatial skills are the most important ones for parametric design. Students learn this on the course "Descriptive Geometry", and special courses and workshops are available in many universities [43, 44].

The knowledge what this subject should cover and deliver to students are can be divided into three main fields. The first of them is the application of parametric design software. The second main field of knowledge is parametric design thinking, which is the cognitive model of the design process [14, 20]. Finally, there are some theoretical knowledge from different scientific fields, mostly from mathematics and computer science, which is necessary for students to be able to apply and understand parametric design, but the architectural curriculum does not cover them. The education method is based on the combined teaching of this three main fields, and the parallel teaching of practical and theoretical knowledge.

7.3 PARAMETRIC DESIGN THINKING AND USING PARAMETRIC DESIGN TOOLS

Architects, engineers and designers still keep changing the definition of "Parametric design thinking". According to Oxman, this change is fundamentally based on the changing and evolving parametric design tools, and on the ways how designers use them. [14] This means that how the tools and abilities of designers change it changes their definition of Parametric design thinking. The same process is noticeable by the students, who start to learn parametric design. As they use parametric design tools and practice the application of parametric software their ability to understand and apply parametric design thinking evolves. Because it lays out the basics, it is of extreme importance for students to be able to apply parametric design thinking, but it is impossible without using parametric design tools. Teaching the application of parametric design tools without the theory of parametric design thinking is possible but pointless, and it results that students can use the tools themselves but they cannot exploit the opportunities of this design tool. This demonstrates clearly the close connection and inseparability of using parametric design tools and parametric design thinking, and shows that the teaching of the application of parametric design software and the theory of parametric design thinking have to be parallel, continuous and related, as it can be seen in Figure 7.1.

7.4 COGNITIVE MODELS: ARCHITECTURAL DESIGN VS. PARAMETRIC SCHEMA

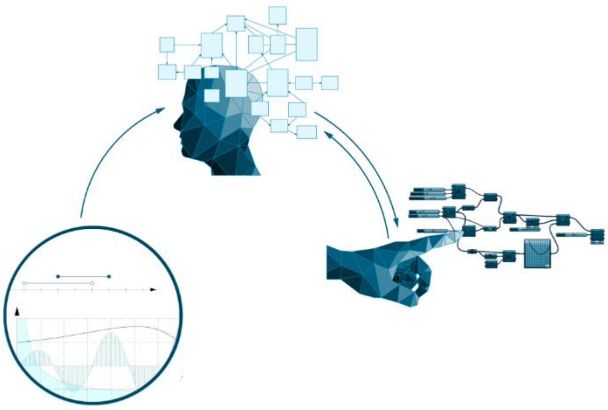


FIGURE 7.1. PARAMETRIC DESIGN THINKING, PARAMETRIC DESIGN TOOLS AND OTHER NECESSARY KNOWLEDGE

Students learn the way of design thinking in each and every semester of the university, for years before they know parametric architecture. They learn how to use their creativity to solve architectural problems and create spaces and buildings. They learn the method of evolution, reflection and re-editing and they identify it as a creative and impressive process. At the University of Pécs students are not allowed to use computer software for architectural design in the first two years, because teachers want to avoid, that the rules and boundaries of these software limit the creativity of the students. They solve every task with the modification and adaptation of their typological knowledge [14].

By using parametric design and making the parametric scheme of a building they have to design a rule structure [14], and for the first sight it seems to be the complete opposite of what they learned for years. Students have to think in rules, formulas and schemes, as it can be seen in Figure 7.2., and it looks like that this design process neglects creativity.

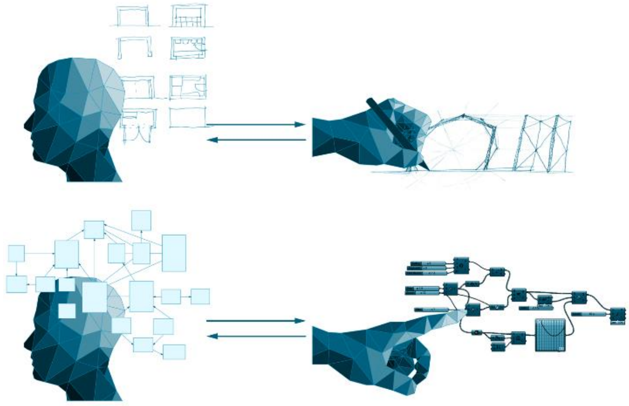


FIGURE 7.2. COGNITIVE MODEL AND DESIGN PROCESS OF TRADITIONAL ARCHITAECTURAL DESIGN AND PARAMETRIC ARCHITECTURAL DESIGN

However, the creative process of parametric design is the creation of the rule system, the parametric scheme itself. The first step of teaching parametric design thinking is the demonstration, that the design effects the rules and not the rules effect the design. During this course the students have to learn how to design a rule-based system instead of an experiment-based design, what they learned to create in the previous semesters [14].

7.5 EDUCATION METHOD

The setup of the schedule of this subject is separated into two segments or half-semesters. In the first one students learn the theoretical and practical basics of parametric design. They build a parametric model together at every class, and every one of these models is destined to demonstrate specific problems and solutions. In the second part of the semester students solve an individual task on their own or in pairs. It helps they engrossment in parametric design, tests and evolves their ability of parametric design thinking.

7.6 EXERCISE MODELS

The education method is based on the combined teaching of practical and theoretical knowledge. Students attention is usually reduced, if they have to learn a bigger amount of theoretical knowledge at the same time, especially, if they think, it is not necessary for they practical work. Moreover, architectural students are often afraid of mathematics. Because of these it is not effective to teach the theoretical knowledge first, and practice later. For this reason, during every class the students make a model, as it can be seen in Figure 7.3. and 7.4., they learn how to use the software and meanwhile they learn, how to solve a specific type of problem, and what theories are behind them. The selected exercises rely on examples from the book of Arthuro Tadeshi [3] and from various internet resources.

Every exercise is started with the determination of the goal. It is a very important step in the education process, because it helps students to think on the solution and improve their skills of parametric design thinking. During the presentation of the exercises the exhibition of parametric design thinking is continuous. Before the start of the exercise solution it is necessary to define the bigger logical steps of the solution. Making this together with the students helps them to acquire this type of solution searching method. Without the targeted teaching of parametric design thinking students often just copy the exercise, and they cannot develop an overall image about the current problem and solution.

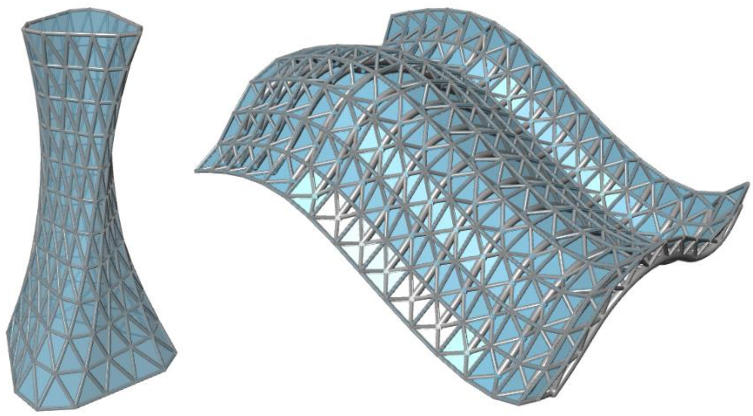


FIGURE 7.3. CLASS TASKS. ROTATING TOWER AND FREEFORM TRUSS-GRID STRUCTURE

After defining the tasks, the production of the actual solution is the next step. This is the part of the exercise, where students learn the practical knowledge, how to use the software and its particular commands. On most classes there is a part of the exercise, where theoretical knowledge is necessary, which students never learned before or they already forget it. This theoretical knowledge can be come from the fields of graphs, intervals or data management. In this case the theoretical education appears like a part of the practical problem solving. Students learn the new knowledge and they immediately learn, how to use it.

Some of the class tasks are architectural examples, like the rotating tower and a freeform truss-grid structure, as it can be seen in Figure 7.3. During making the rotating tower students learn how to use series, domains, graphs and the data management, and to design the truss-grid structure they have to use meshes and understand how the surfaces' UV-coordinates are used. Other exercises introduce useful design solutions, which are specific to parametric design, like force field and Voronoi diagram as it can be seen in Figure 7.4.

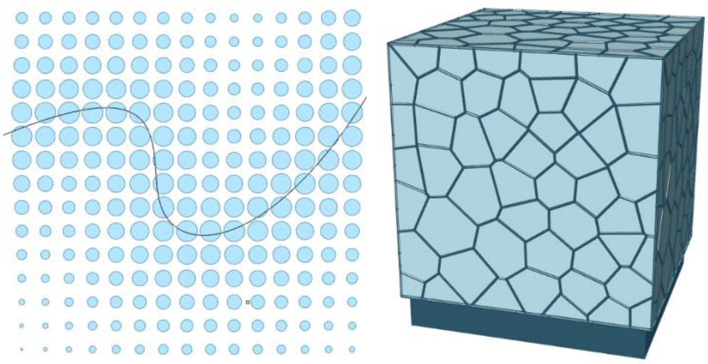


FIGURE 7.4. CLASS TASKS. FORCE FIELD AND VORONOI DIAGRAM

7.7 INDIVIDUAL TASKS

After the sample exercises the students have to accomplish an individual design task, which helps them to learn the creative process of creating a parametric schema. Choosing the task itself is a challenge for students. The selection of a right exercise indicates clearly if the students understand parametric design thinking and if they can recognize in which tasks can it be helpful for them. During the design process the students can experiment and evolve their ability of parametric design thinking, they learn to use the right tools, and utilize their possibilities. As their individual task students can create complex building designs, using their previously learned architectural design knowledge and the newly acquired parametric design thinking together as it can be seen in Figure 7.5. and 7.6.

FIGURE 7.5. STUDENT'S WORK. MADE BY DALMA LOVIG AND ÁKOS KARANCZ

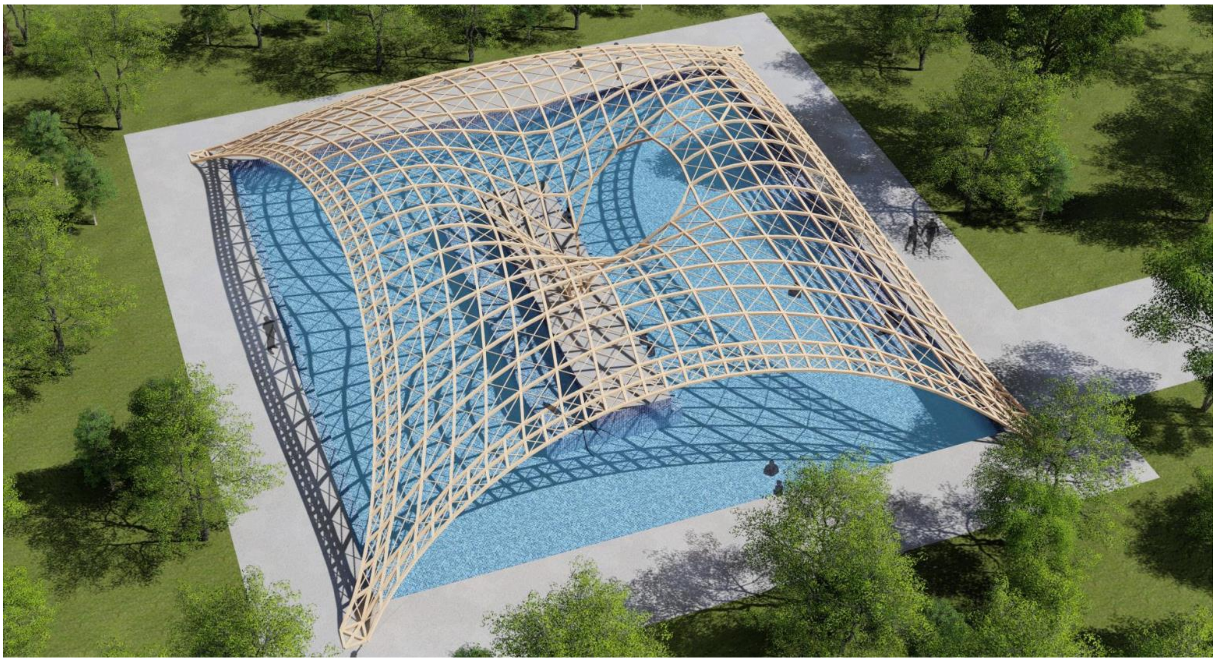


FIGURE 7.6. STUDENT'S WORK. MADE BY KINGA ZSÓFIA HORVÁTH, ERIKA LUTHÁR AND GÁBOR SZARVAS

7.8 MATERIALIZING PARAMETRICS WORKSHOP

As the continuation of the “Parametric constructional design” course a workshop called “Materializing Parametric Design” was held on March 6.-7. 2017, with the help of Bojan Tepavcevic from the University of Novi Sad.

The workshop was created for students who are already familiar with parametric design. It meant to represent methods for creating real-life models of parametric structures. Parametric design demands at least partly automatized manufacturing. The students learned the techniques which can be used to create smaller scaled models. As the result of the workshop students are able to prepare a model for laser cutting or 3D printing.

The model for the workshop was created together with Bojan Tepavcevic, who has routine in digital fabrication [45]. The students learned how to create the model on a class of an elective parametric design course. A lot of students attended this as the second semester of parametric design. On the workshop students learned how to fold out the model, and then they created the model from the printed elements, as it can be seen in Figure 7.7. They also learned the basics of architectural visualization.

They also learned how to prepare models for 3D printing from István Háber [46], and they were able to follow the process of 3D printing with the help of the 3D Print Facility of the University of Pécs [47].

FIGURE 7.7. WORKSHOP PHOTOS

The created models can be seen in Figure 7.8.

FIGURE 7.8. 3D MODEL AND PAPER MODEL CREATED ON THE WORKSHOP

7.9 SUMMARY AND CONCLUSIONS

The introduced teaching method leads students into the world of parametric design. They learn the basics of software application and parametric design thinking through the class exercises. During the creation of an individual design task students can dive in the parametric design. Students are interested in this new field of architecture and they see the new possibilities of this evolving design tool. After the course the students are able to create a simpler parametric schema and they possess the knowledge to be able to improve their parametric capabilities.

7.10 CASE STUDIES

During the research and teaching parametric design some parametric models were made for numerous purposes. These are visible in the following. These represent clearly some specific aspects of learning and teaching parametric design.

7.10.1 ARCHICAD-GRASSHOPPER CONNECTION PROJECT

This project was made to present the possibilities of the ArchiCAD-Grasshopper connection software [48] for the Grasshopper-ARCHICAD Workflow Competition announced by Graphisoft. The winners were announced in August 4, 2016 and this project happened to be a runner-up winner. It consists of two parts. The first part explains the advantages of utilizing parametric design in the design process of conventional buildings. The second one explains the possibilities of this method in shape creation. Furthermore, this project contains suggestions for the development of the AC-GH connection. It presents that the cooperation of the two programs in some specific fields would be beneficial. For this project ArchiCAD 19 [49], Rhino 3D [15], Grasshopper [16], and the plug-ins of Grasshopper, named LunchBox [50] and Kangaroo [51] were used.

7.10.1.1 PARAMETRIC DESIGN IN CONVENTIONAL ARCHITECTURE

In this project an office building was created, as you can see in Figure 7.9. The building was created to fit to more building sites. The position of front and side walls are linked from ArchiCAD, and the back wall is always parallel with the front wall. The width of the building, and the height of the floors is defined with a variable in Grasshopper, can it can be changed freely. The floor plan changes according to the size of the building.

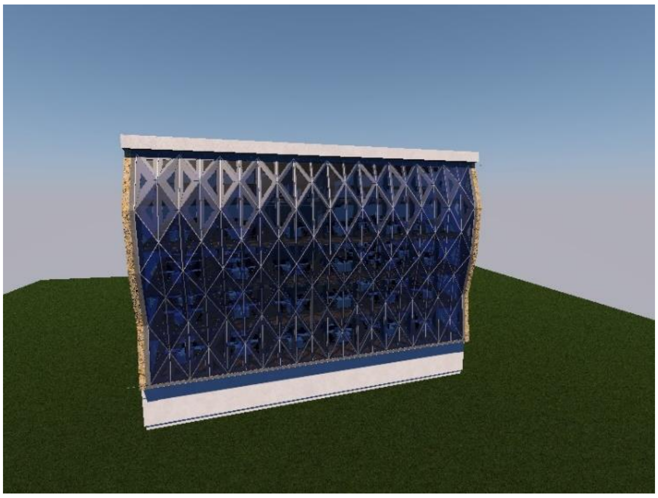


FIGURE 7.9. PARAMETRIC CURTAIN WALL

7.10.1.2 USING PARAMETRIC DESIGN TO CREATE UNIQUE SHAPES

This part of the project is a unique curtain wall. In this stage of the software development it was not possible to create it with the curtain wall tool, because it cannot create this unique shapes. This changed since then and the newest version of the software contains a solution for special curtain wall elements. The creation of this structure would be quite a challenge with traditional tools, however, it can be created in Grasshopper easily.

7.10.2 IDEA COMPETITION FOR THE ARCHITECTURAL AND URBANIST VISION OF THE 2024 OLYMPIC GAMES IN BUDAPEST

The competition was announced by Budapest2024 Nonprofit Zrt. in July 1. 2016 and the prices were announced in March 7. 2017. The plan earned 2nd price.

The main element of this project was a stadium deigned with parametric tools. It was a very exciting task, because the stadium has to be able to fit 60 000 persons during the Olympics, but after that it has to be downsized to 15 000 persons. For this reason, the biggest part of the grandstand is removable, and the outer wall is made from self-supporting panels, where the middle elements can be removed, and the outer ones can be replaced for the modification, as it can be seen in Figure 7.10, and 7.11.

The urban design was created together with Miklós Kelenffy and Polett Óvári, as it can be seen in Figure 7.12.

Parametric design was used to calculate the size and the shape of the grandstand. The calculation of a grandstand has to be made row-by-row, so every people will be able to watch over the head of the person who sits before them [52]. This calculation was created in Grasshopper, and it showed, that the same grandstand can be thinner and higher or thicker and lower to create adequate visibility. In this case a lower, wider grandstand was used, so the structure of the roof can be lower, which is useful after the transformation.

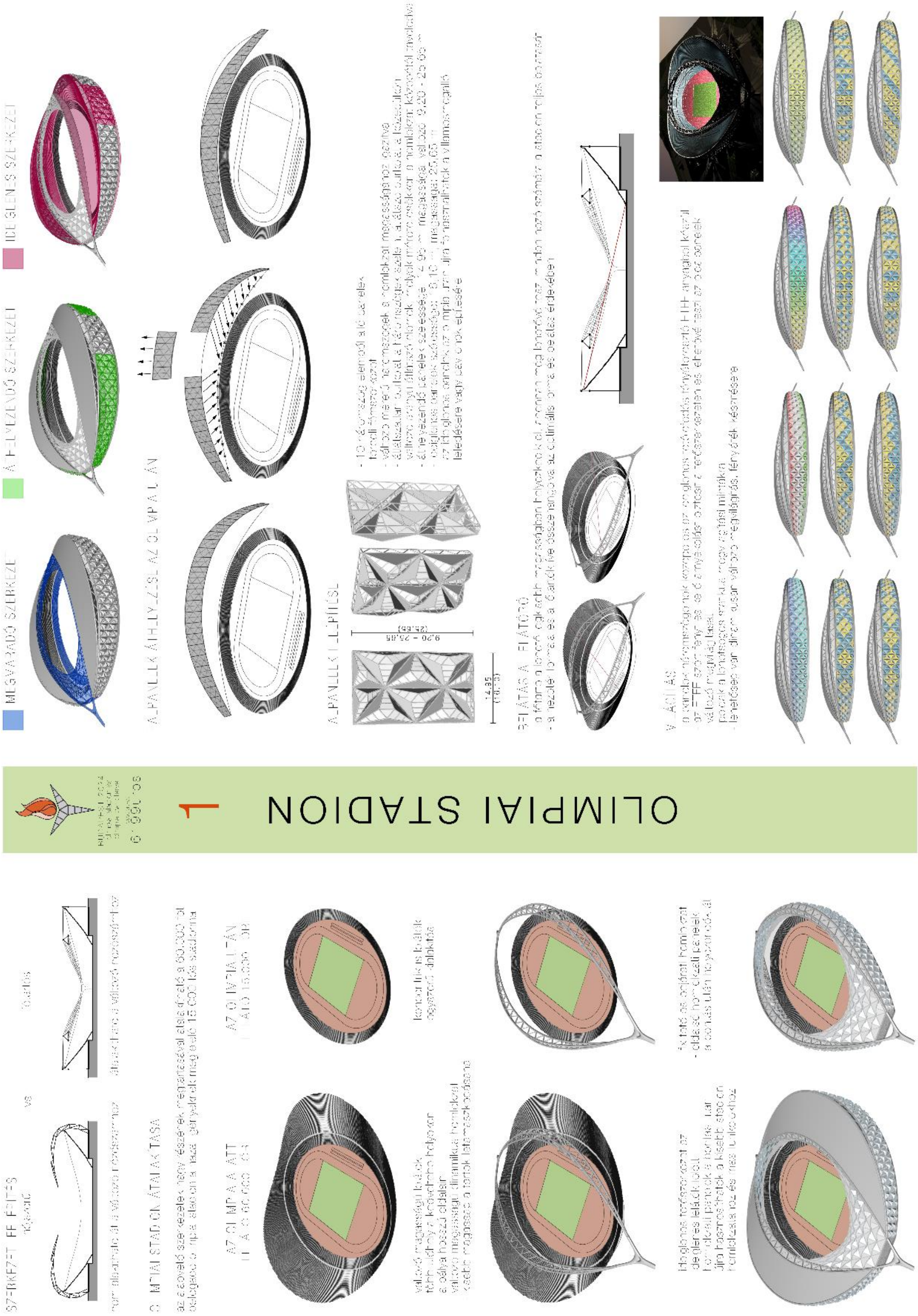


FIGURE 7.10. ARCHITECTURAL CONCEPT

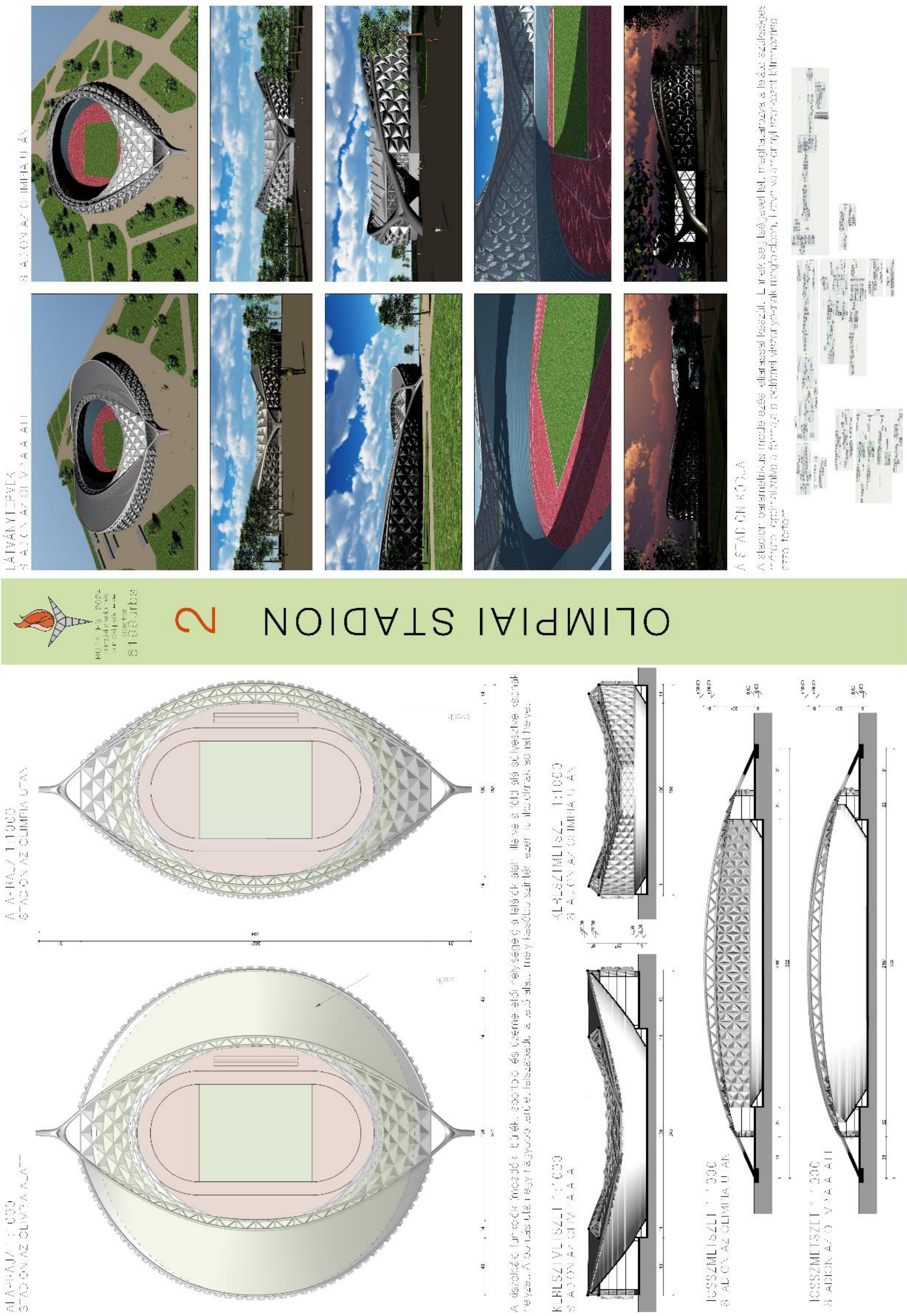


FIGURE 7.11. FLOOR PLANS, SECTIONS, PERSPECTIVE VIEWS

FIGURE 7.12. URBAN PLAN

7.10.3 ANAMNESIS

This sculpture plan was created by Zoltán Pál as a part of the project "Millenáris Széllkapu" in Budapest, as it can be seen in Figure 7.13. Because of its big size a more efficient manufacturing solution was necessary, which resulted the triangulation of the sculpture. For the triangulation and later for the construction plans parametric design was a very powerful tool.

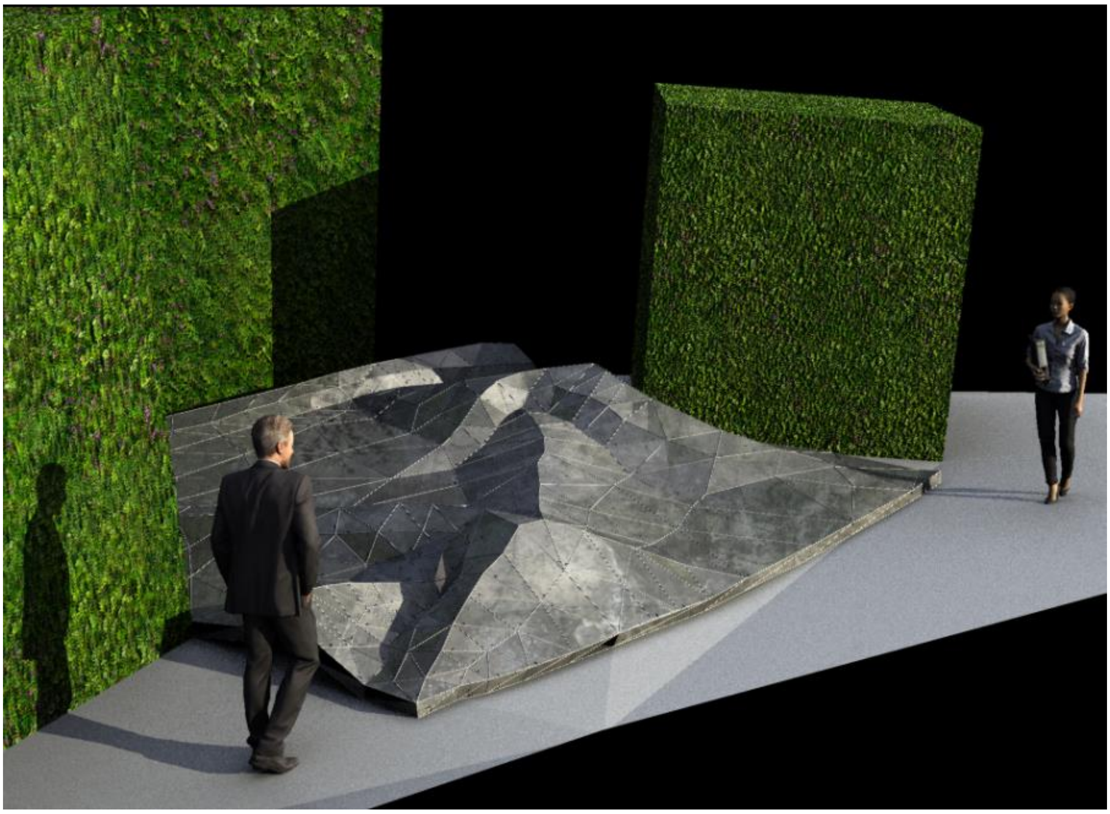


FIGURE 7.13. VIZUALIZATION OF THE SCULPTURE

The sculptor created a model from lead, which was 3D scanned. The points for the triangular grid were placed to this model in Rhino 3D, and the triangular grid was created with Grasshopper. This method required the strong connection between the two software, and because the triangular grid was always recalculated by Grasshopper, the design process was much quicker than usually.

Because of the lot of different elements, the construction plans – a part of it is visible in Figure 7.14. – had to be created with parametric tools too.

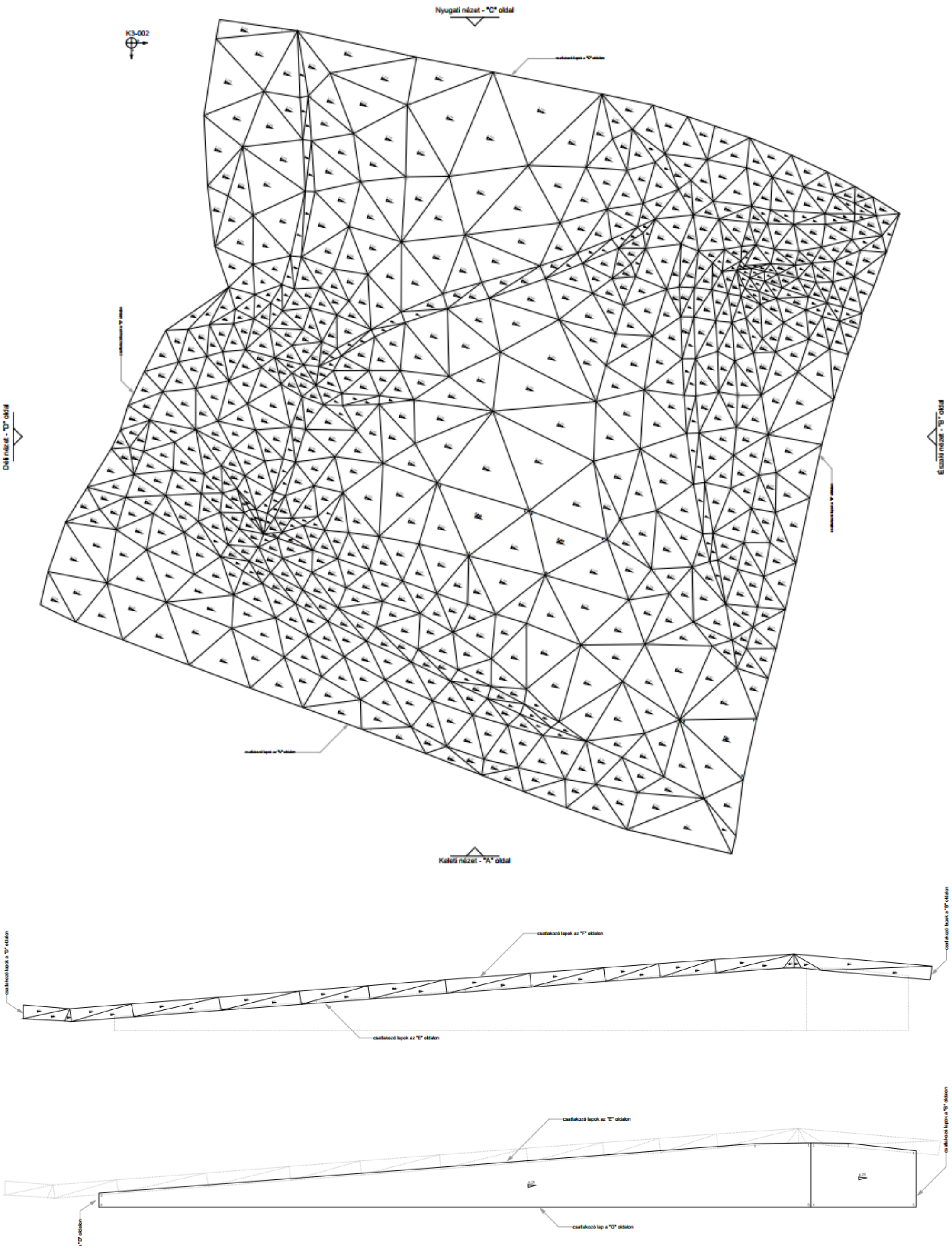


FIGURE 7.14. CONSTRUCTION PLANS

8 CONCLUSIONS

In this research the parametric design techniques were analyzed and classified. To be able to do this it was necessary to understand the logic, the cognitive model of parametric design. How a shape or a pattern is created depends on the shape or the pattern itself. The different surfaces, even freeform surfaces are well researched in the field of geometry, so the main focus was the creation of the patterns. It was necessary to examine patterns on a more comprehensive way. For this the graph representation of patterns were used. Based on the graph representation of patterns they were classified as regular and irregular patterns, which is the base of the classification system.

A dome and vault creating tool was developed for parametric design, which basic concept relies on Formex Algebra. This tool creates spherical, cylindrical or polar grids from rectangular or triangular grids with coordinate system transformation. A similar tool was created in a more user friendly environment, with a newly developed calculation method for easier application. The environment is Grasshopper, which is a graphical algorithm editor and compared to Formex Algebra it does not require programming skills from users.

A similar, but more freeform tool was created for rotational grids. Two different versions were developed. The first one is a modified version of the cylindrical tool, where the second direction is vertical, the thickness of the grid is horizontal. The second coordinate direction of the second tool is alongside the generating curve, and the thickness is always perpendicular to the curve.

A series of domes were created with the previously introduced dome tool. Single-layer truss-grid domes, single-layer domes with fix joints and double-layer truss-grid domes were created and analyzed. The graph characteristics of the domes were calculated. It showed that graph characteristics are correlated with the statical behavior of truss-grid domes. The graph characteristics of single-layer truss-rid domes are in clear correlation with their graph characteristics. In the case of double layer truss-grid domes the correlation is not clear, some exceptions appear, but still can be used to improve the build-up of a designed dome structure.

A syllabus was created for architectural students with which they are able to acquire parametric design easily and effectively. To aim this goal, the earlier results were used especially from the first part of the research. The knowledge of the cognitive model of parametric design and the parametric design tools is the first step to define an efficient teaching method. The necessary knowledge has to be also defined and examined which part of this knowledge is already thought for architectural students and which has to be covered by this subject. With the consideration of all of these factors a syllabus was defined which makes possible to implement parametric design to the curriculum of architectural students.

During the research numerous connecting design tasks was also made. The theoretical knowledge earned during the research constituted a big advantage during the design tasks. At the same time, the practical solutions earned during the design tasks helped to understand parametric design, its tools and cognitive schema much better.

9 Acknowledgements

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