University of Pécs Faculty of Sciences Doctoral School of Earth Sciences

# The use of mathematical models in the analysis of economical and geographical problems, on the example of the Trans-Danubian region

**PhD-dissertation theses** 

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# **1. Introduction**

With the evolution of information systems, the results of geography and the results from all fields of different disciplines become more and more entangled, finding ever more common applications. This process is especially spectacular with natural sciences like mathematics, statistics and computer sciences. In connection with geography, the most important role is of geographic information systems; many aspects of which may be regarded as geographical applications of the mathematical sciences. Geo-informatics, as it names shows, regards informatics systems, where the results of the fields of geography serves as input data.

Geo-informatics systems offer several advantages over more traditional approaches. The most common is that it represents data and information by the means of figures and maps, significantly simplifying and speeding up their understanding and analysis. Similar, maybe a bit less spectacular bit nevertheless not less important is the drastic increase of the amount of data that is possible to be handled, overviewed and analyzed. Modern informatics systems and mathematical models both have increased the amount of process able data by magnitudes by themselves, while their joint applications have already proven in many everyday applications.

The thesis, driven by geographical data (Trans-Danubian Region) and following a problem-solution approach, presents research and data analysis regarding the development and applications of mathematical methods for geographical related problems. This approach provides basis the increase of the accuracy of the way geographical (and other) factors are incorporated into the calculations; the methods require sophisticated and multi level data

input, which is more labor and cost intense and demands higher levels of preparation than the traditional, general practice.

Mathematical models and methods gained a significant role in the fields of economy strategy and in development planning during the last decades. The thesis provides a summary of geo-informatics related mathematical models, shows it wide range and diverse applicability, while presenting several numerical case studies. Just as in the case of geo-informatics, in the case of mathematical modeling a balance must be found between a model that best models and approximates the problem at hand, while also keeping it solvable in given time frame using the available hardware, and keeping it possible to be populated with all required data.

The models are presented in a general form, although for each concrete applications, the computations are always based on data from the Trans-Danubian Region. From the disciplines of mathematics, the thesis aim to cover a wise range of different fields, including methods from statistics, operations research, data mining and machine learning.

The summary of the main and special characteristics of the Trans-Danubian Region is also part of the thesis. This is especially important for the development of correct mathematical models, and from the aspect of analyzing the results and drawing the right conclusions. From a sufficiently large and diverse data set, a good statistician can virtually draw any conclusion using the right method and parameters. Thus the mathematical methods alone are not necessary; for their proper application and analysis, a deep understanding of local problems and relations are also necessary.

### Reasons behind the Trans-Danubion region falling behind

From the comparison with the nationwide data and from the special location of the area, the thesis draws the following conclusions. Due to its southern border location, the region gets significantly less development resources. The population is decreasing; the demographical tree is shifting towards the elderly, increasing the maintenance cost of the social network. Three quarters of the settlements consists of small towns or hamlets, the regional centers fail to offer complete functionality; in other words, the structure lacks a city structure that would serve as regional centers. This contributes to the lack of job opportunities, and induces significant commute (about 30%).

The number of disadvantaged areas within the region is higher than the national average. Unemployment rates and economical inactivity is high. Economical performance if low, the export ratio is small, and mainly due to the under developed-infrastructure (especially the road system) foreign investments rates are also low.

Tourism is seasonal (with the Balaton area being dominant) and the number of higher standard accommodations is few. The region falls far from the regionally determining Wien-Budapest line.

# 2. Aims of the thesis

To develop quick mathematical and optimization models serving as decision support systems specifically localized for the Trans-Danubian region aiming to help coping with the financial crisis started in 2008. The main goal is thus to create and demonstrate on examples models based on mathematical statistics and optimization methods.

An important aspect of modeling is the easy applicability and deploy-ability, aiming to be based on tools that are already widely available or easily attainable on everyday computer systems.

### **Placing the thesis**

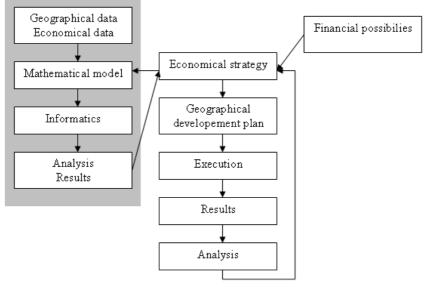


Figure 1. Placing the thesis. (CSIZMADIA G. 2009.)

## **3.** Methodology and source of data

The models developed in the thesis use a wide range of mathematical methods, mainly from the following three disciplines: mathematical statistics, operations research and data mining & machine learning.

The majority of the presented models, using the given data resolution levels during the computations (factors taken into account and number of time intervals) do not exceed to the

possibilities of the statistical functionalities of Microsoft Excel; furthermore, although not as well known, but the optimization models can be handled by the built-in Excel Solver (despite the serious size limitation of 200 decision variables). The relative simplicity of the methodology makes it possible to solve large scale real life problems on today's widespread platforms. This is a very important aspect, since this means that in order of development, support and solution of the localized version of the model, there is no need for special hardware and software background, research and viability can be assessed using Excel which is essentially available on every computer. The solution of the final, more complex and large scale model might require specialized optimization software.

#### The modeling process

The thesis presents the complete process of mathematical geomodeling.

Although the data collection phase might seem a relatively simple task, it is often a surprisingly demanding and time consuming task, with often several implications to the modeling step as well.

During the modeling process, it might seem natural that the after determining the information requirements of the model, as a first step of implementation we acquire the necessary data. In practice however, often the data available/acquirable determines the kind of model we build to solve a particular problem. After the data collection based on the annual books, reports and correspondence with the Hungarian Statistical Agency (KSH), several data tables proved to be incomplete; in such cases, the thesis aims to substitute the missing data sequences with related data sequences, or to make the models independent from them.

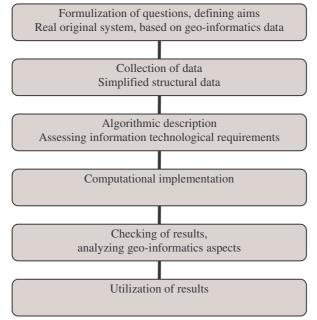
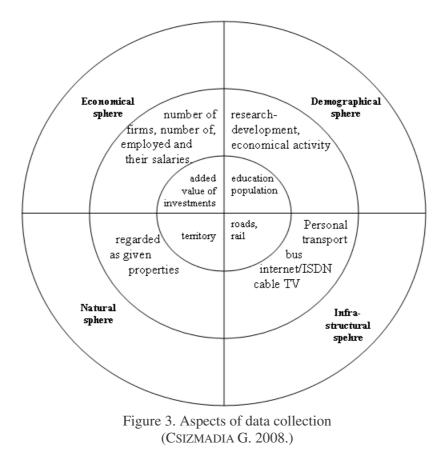


Figure 2. The process of mathematical geo-modeling (CSIZMADIA G. 2008.)

The collected data served as bases and proof for the conclusions and models of the basis. During the collection of data, regarding the main spheres of interest, I concentrated on the following areas (*Figure 3*):



## 4. Main results

The first and most resource intensive step of model building is the data collection, identifying the overlapping area of those necessary in an optimal case and those that is acquirable. Although model building and data collection mutually rely on one other, the natural approach puts data collection first nevertheless.

Modeling related to unexpected events mainly regard systems and resources of strategic importance. Transportation systems like rail, roads, waterways and airports, public services like electricity, gas and drinking water and most communications system can all be modeled as networks. Active capacities like the available manpower and available power machines of the regional catastrophe recovery and other enterprises, but also passive resources like available stocks or storage capacities provided by larger firms and supermarkets can all be modeled as resource allocation and management models. In both cases, priorities typically shift towards more densely populated areas. Assessing the importance and weights of the different modeling factors are usually also part of the data collection step.

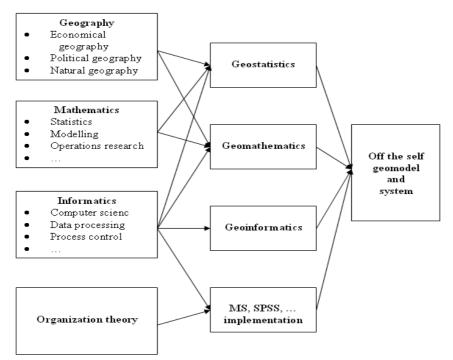


Figure 4. Main components of modeling and applications.

#### (CSIZMADIA G. 2008.)

The developed models mainly rely on the methods of mathematical statistics for forecasting, while for solving problems regarding networks, optimal resource allocation the models are based on methods from operations research, paying special attention to algorithms for networks, resource usage optimization and methods for forecasting and analyzing trends.

The thesis builds models for several applications. In the following we provide only a summary of these models due to size limitations; however, as a demonstration I present here a concrete model regarding development strategies in more detailed with actual numerical results for the Trans-Danubian region.

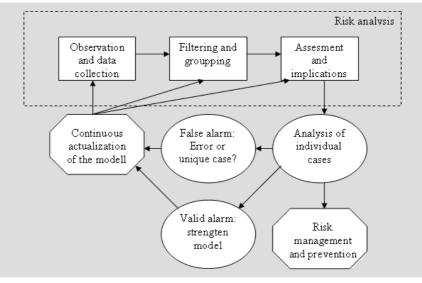


Figure 5. The process of risk analysis. (CSIZMADIA G. 2008.)

After presenting a model for assessing the components of human resources – an example for a project based approach – the thesis develops mathematical modeling possibilities for the analysis of the economy and for fast decision support for the Trans-Danubian region.

The methods of analysis depend on the kind of answers expected. Online type decision support (few seconds or less, like for checking bank transactions) usually rely on methods based on rules. A common property of such systems that they are trained on non real time historical datasets, creating a rule system that accepts data that harmonizes with the training set, and reject outliers (a typical such system are the decision trees).

In case of offline type decision support (in which case there is significant time available for analysis) it is often more reliable to apply models that are not based on predetermined rules (e.g. pattern recognition type algorithms).

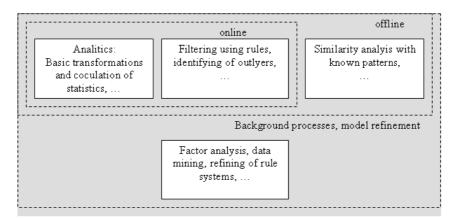


Figure 6. Layers of the analysis methodology. (CSIZMADIA G. 2008.)

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In case of offline analysis it is important to emphasize the difference between risk analysis and risk management. The aim of the analysis is to filter, group and preprocess the data, as in this case there is a possibility for deeper analysis.

Naturally, data collection is typically followed by simple statistical analysis before the data could serve as input for more complex models. I analyze and provide forecast for such important factors like the change of population or the industrial output of a region, the number of enterprises with foreign interest or the percentage of their foreign ownership, or to the change in economical activity.

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65-         éves         -007%         -001%         0.37%         0.65%         1.00%         1.20%         0.87%         19.79%         1.23%         3.06%           Közép Dunantul         1.35%         0.33%         0.65%         1.00%         1.20%         1.29%         26.91%         1.23%         3.06%           Nyugat-Dunantul         0.45%         0.03%         0.58%         1.10%         1.10%         1.29%         26.91%         1.62%         1.20%           DelJunantul         0.68%         0.73%         0.65%         0.58%         1.00%         1.20%         1.22%         26.01%         1.62%         3.06%           DelJunantul         0.68%         0.73%         0.65%         0.58%         0.26%         0.22%         0.42%         15.44%         0.44%         0.54%         1.00%         1.92%         0.24%         15.44%         0.47%         100.44%         5.44%         0.47%         100.44%         5.44%         0.47%         100.44%         5.44%         0.47%         100.44%         5.44%         0.67%         0.22%         0.45%         0.25%         0.45%         0.26%         0.45%         0.26%         0.26%         0.26%         0.26%         0.67%         0.27%	9 C	el-Alföld	-0,26%	-0,24%	-0,30%	-0,15%	-0,38%	-0,20%	-0,25%	-0,17%	71.60%	-0,31%	30,59%	
Közép-Magyarotszág         -0,07%         -0,01%         0,37%         0,65%         1,00%         1,20%         1,44%         0,87%         19,79%         1,23%         3,06%           Közép-Dunantul         1,35%         0,33%         1,31%         1,23%         1,65%         1,44%         0,87%         26,91%         1,62%         12,20%         26,91%         1,62%         12,20%         26,91%         1,62%         12,20%         26,91%         1,62%         12,80%         1,00%         1,98%         0,58%         5,181%         1,04%         5,48%         1,00%	0 0	Irszágösszesen	-0,01%	-0,19%	-0,08%	-0,05%	-0,11%		-0,05%				48,72%	
Isozép-Dunántil         1,35%         0.83%         1.31%         1.22%         1.65%         1.41%         1.29%         2.691%         1.62%         1.28%           Nyuga-Dunántil         0.46%         0.80%         0.66%         0.59%         1.10%         1.10%         1.12%         1.29%         2.691%         1.62%         1.24%         5.48%           Del-Dunántil         0.66%         0.73%         0.67%         0.59%         1.10%         1.10%         1.98%         0.89%         51.81%         1.04%         5.48%           Del-Dunántil         0.66%         0.73%         0.67%         0.59%         0.58%         0.62%         0.44%         15.44%         0.04%         0.52%         0.44%         15.44%         0.04%         0.62%         0.74%         0.67%         0.41%         0.56%         0.56%         0.42%         0.27%         11.49%         0.22%         28.40%           Eszak-Alfold         0.62%         0.73%         0.64%         0.66%         0.64%         0.66%         0.64%         0.66%         0.64%         0.66%         0.64%         0.66%         0.64%         0.66%         0.64%         0.66%         0.67%         3.79%         0.64%         0.67%         3.79%	1 (	55-éves												
IKożej Dumântići         1,36%         0,83%         1,31%         1,22%         1,62%         1,23%         1,23%         2,631%         1,22%         1,28%           Nyugat Dunántići         0,46%         0,86%         0,86%         0,59%         1,10%         1,10%         1,98%         0,84%         1,04%         5,48%           Deł Dunántići         0,68%         0,74%         0,67%         0,41%         0,58%         0,58%         0,26%         0,44%         1,54%         1,04%         5,48%           Deł Dunántići         0,68%         0,77%         0,41%         0,58%         0,58%         0,28%         0,44%         1,54%         1,04%         5,48%           Eszak-Alfold         0,67%         0,41%         0,58%         0,58%         0,22%         0,44%         1,54%         0,22%         28,40%           Deł Alfold         0,26%         0,73%         0,25%         0,43%         0,25%         0,47%         0,07%         0,95%         9,97%           Deł Alfold         0,26%         0,37%         0,56%         0,56%         0,27%         0,56%         0,56%         0,97%         0,56%         0,97%         0,66%         10,00%         0,05%         0,97%         0,56%	2 K	özép-Magyarország	-0.07%	-0.01%	0.37%	0.65%	1.00%	1.20%	1.49%	0.87%	19,79%	1.23%	3.06%	
Del Dunántůl         0.68%         0.73%         0.41%         0.58%         0.58%         0.92%         0.44%         15,14%         0.47%         0.87%           Eszak Alfold         0.62%         0.74%         0.67%         0.49%         0.39%         0.25%         0.27%         11,19%         0.22%         28,40%           Eszak Alfold         0.62%         0.73%         0.29%         0.64%         0.22%         23.40%           Diel-Alfold         0.62%         0.73%         0.62%         0.24%         0.27%         0.49%         0.25%           Diel-Alfold         0.62%         0.73%         0.62%         0.50%         0.71%         0.76%         0.73%         0.64%         10.80%           Országiosszesen         0.45%         0.47%         0.60%         0.67%         0.91%         0.91%         0.90%         0.67%         3.79%			1,36%	0,83%	1,31%	1,32%	1,66%			1,29%	26,91%		12,80%	
Eszak-Alfold         0.74%         0.87%         0.29%         0.33%         0.43%         0.25%         0.27%         11.49%         0.22%         28.40%           Eszak-Alfold         0.62%         0.73%         0.29%         0.85%         0.22%         0.33%         0.49%         0.20%         0.49%				0,80%					1,19%				5,48%	
Eszak-Alfold         0,62%         0,73%         0,29%         0,66%         0,64%         0,22%         0,33%         0,49%         20,07%         0,58%         49,97%           Del-Alfold         0.28%         0.37%         0,52%         0,50%         0,71%         0,56%         0,49%         0,49%         0,07%         0,58%         49,97%           0 forzágósszesen         0,45%         0,47%         0,50%         0,57%         0,83%         0,91%         0,70%         19,90%         0,87%         3,79%           Atragos hiba:         24,79%         24,13%         24,13%         24,13%         24,13%         24,13%														
Dél-Alfold         0,23%         0,37%         0,52%         0,57%         0,71%         0,75%         0,75%         0,75%         0,64%         10,80%           Országiosszesen         0,45%         0,47%         0,50%         0,57%         0,83%         0,91%         0,90%         0,70%         19,90%         0,87%         3,79%           Artagos hiba:         24,79%         24,13%         24,13%         24,13%         24,13%	6 É	szak-Magyarország	0,74%	0,67%	0,49%	0,39%	0,35%	0,43%	0,25%	0,27%	11,49%	0,22%	28,40%	
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Atlagos hiba: 24,79% 24,13%	3 D	el-Alföld	0,28%	0,37%	0,37%	0,62%	0,50%				27,36%	0,64%	10,80%	
	9 0	rszágösszesen	0,45%	0,47%	0,50%	0,67%	0,83%	0,91%	0,99%	0,70%	19,90%	0,87%	3,79%	
	0									Átlagos hiba:	24,79%		24,13%	

Figure 7. Snapshot of the checking calculations. (CSIZMADIA G. 2009.)

I test and check my models and computational results by the means of using it to predict and check against known data. The average

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forecast error is around 24%, which - taken into account the rather short analyzed period, the data and factors that effect the model but can't be mathematically modeled – is regarded as acceptable. The figure above provides a snapshot of the testing process.

### Applying statistical models of forecast determining the main future sources of endangerment in the Trans-Danubian region

In the following I present a model and the related actual numerical results in detail. The main aim of the model is to draw attention to the unfavorable economical forecasts of the region through the means of statistical and operation research models.

It's hard to accurately define the factors determining the development of a region. Using statistical methods and solution methods from operations research, I determine an approximation function defining the development of a region based on the main regional factors. As a general indicator of development, I chose the GDP relative to population. As learning set serving as basis of determining the approximation, I used the following main factors:

- A. GDP (million forints, per thousand people)
- B. Economical activity (per thousand people)
- C. Number of registered firms (per thousand people)
- D. Ratio of foreign capital in firms with foreign investors (million forints, per thousand people)
- E. Average gross salary of employees, physical work
- F. Average gross salary of employees, professionals
- G. Number people employed in research and design (per thousand people)
- H. Cost of research and development (million forints, per thousand people)
- I. Number of telephone lines (per thousand people)

J. Length of all main roads (motorways and main country connections), relative to area

Using the quadratic deviance function as a measure of approximation error, the determination of the approximation function is a minimum squares problem. The value of the error variables  $\mathcal{E}$  can be arbitrary. On the other hand, we assume the non-negativity of the coefficients B,C,...,J. There are two reasons for this. First, from the correlations with the GDP values we see that the relation is positive, and on the other hands, without such assumptions the models tend to over-optimize the problem; thus the inclusion of background knowledge yields a favorable model.

$$\begin{split} \min \mathcal{E}_{Pest}^2 + ... + \mathcal{E}_{Csongrád}^2 \quad & (\text{so the approximation error is minimal}) \\ Bx_{Pest(B)} + ... + Jx_{Pest(J)} + \mathcal{E}_{Pest} = GDP_{Pest} \quad & (\text{approximation for Pest}) \\ & \dots \\ Bx_{Csongrád(B)} + ... + Jx_{Csongrád(J)} + \mathcal{E}_{Csongrád} = GDP_{Csongrád} \quad & (\text{for Csongrád}) \end{split}$$

The values of the coefficients of the development function are presented together with another aspect of the optimization process.

The actual solution values, and part of the numerical model is shown from the screen shot of Xpress 7's student version, shown below.

(http://optimization.fico.com/student-version-of-fico-xpress.html)

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0.9788 B + 0.8374 C + 0.774 D + 0.9886 x + 0.9647 C + 0.1850 D + 0.9983 x + 0.9677 B + 0.9298 C + 1.0000 D + 0.9933 x + 1.0000 B + 1.0000 C + 0.6456 D + 0.94819 x + 0.8664 E + 0.9396 C + 0.0460 D + 0.9819 x + 0.8666 B + 0.9396 C + 0.0461 D + 0.9819 x + 0.7785 B + 0.9396 C + 0.0461 D + 0.9819 x + 0.7785 B + 0.9716 C + 0.1237 D + 0.9856 x + 0.8667 B + 0.5566 C + 0.1237 D + 0.9855 x ± + 0.8667 B + 0.5566 C + 0.1231 D + 0.9352 x ± + 0.8667 B + 0.5566 C + 0.1231 D + 0.9355 x ± + 0.8667 B + 0.5566 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.7565 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1813 D + 0.9355 x ± + 0.8356 B + 0.6689 C + 0.1512 D + 0.9355 x ± + 0.90509 -2.70096-015 -2.70096-015 - 0.90512 -2.70096-015 - 0.90512 -2.70096-015 - 0.90512 -2.70096-015 - 0.9052 D + 0.02519 X ± + 1 0.449037 -6.23986-013 - 0.90512 - 2.0276 -4.56266-015 - 1 0.0004/fnot / %ats Mark Solutions Objective MIP search MP se	0.9788       B +       0.8374       C +       0.9774       D +       0.9886       x E +       FP3       FP3 <td></td> <td>-</td> <td></td> <td></td> <td></td>												-			
0.9544       B +       0.9373       C +       0.1890       D +       0.9895       X +       EPS Tolina       0.055105       1.11329-012         0.9693       B +       0.9647       C +       0.0464       D +       0.9983       X +       EPS Tolina       0.055105       1.11329-012         0.9693       B +       0.9647       C +       0.0464       D +       0.9983       X +       EPS Tolina       0.0252105       1.13372e-017         0.7685       B +       0.99962       C +       0.0460       D +       0.8122       X +       EPS Tolina       0.0145007       6.59195e-017         0.7828       B +       0.7916       C +       0.1372       D +       0.8956       X +       EPS Tolina       0.014507       6.59157e-017         0.7815       B +       0.6569       C +       0.1237       D +       0.8956       X +       EPS Tolina       0.014507       6.59157e-017         0.8055       B +       0.7358       C +       0.1237       D +       0.8928       X +       EPS Tolina       0.0156264       10.59739-017         0.8056       B +       0.7358       C +       0.7318       D +       0.8217       X +       D       0.01	0.9694       B +       0.9973       C +       0.1890       P +       0.9983       X +       Image: C +       0.1830       D +       0.9983       X +       Image: C +       0.05810       1.11323-0.01         0.9683       B +       0.9647       C +       0.0456       X +       Image: C +       0.9466       X +       Image: C +       0.9476       X +       Image: C +       0.9486       X															
0.9677       B +       0.9288       C +       1.0000       D +       0.9993       x +       FFS_BAZ       0.064875       -6.33898-017         1.0000       B +       1.0000       C +       0.6454       D +       0.9819       x +       FFS_BAZ       0.064875       -6.33898-017         0.0464       B +       0.09647       C +       0.0761       D +       0.8819       x +       FFS_Bayed       -0.23324       -2.23045-016         0.7685       B +       0.9374       C +       0.1372       D +       0.0819       x +       FFS_Bayed       0.030262       2.08167-016       FFS_Bayed       0.03267       2.08167-016       FFS_Bayed       0.03276       2.08167-016       FFS_Bayed       0.03267       2.08167-016       FFS_Bayed       0.03276       2.08167-016       FFS_Bayed       0.03214       4.50371-015       0.044159       7.33276-017       FFS_Bayed       0.03214       4.50371-015       0.0328       x F       0.03285       x F       0.03285       x F       0.03285       x F       0.0161658       3.13	0.9677       B +       0.9289       C +       1.0000       D +       0.9933       X +       IF3_BA2       0.0648785       -6.33898-017         0.9687       B +       0.6647       C +       0.6456       D +       0.9819       X +       IF3_BA2       0.0648785       -6.33898-017         0.8646       B +       0.0966       C +       0.0761       D +       0.8819       X +       IF3_BA2       0.0648785       -6.33898-017         0.8646       B +       0.0966       C +       0.0761       D +       0.8819       X +       IF3_BA3       -0.016207       EF3_Hograd       -0.23324       -2.23245       -0.0569787       0.08070       6.99159       D -       0.2378       D -       0.0237       D -       D -       D -       D -       D -       D -       D -       D -       D -       D -       D -       D -       D -       D - <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>															
0.9883       B +       0.6647       C +       0.6466       N +       N         1.0000       B +       0.000       C +       0.0751       D +       0.8963       N +       PS       Heves       -0.0917612       -1.33776-017         0.7805       B +       0.8966       C +       0.0751       D +       0.8512       x E +       PS       Heves       -0.043707       E-3915-017         0.7785       B +       0.7985       C +       0.0237       D +       0.8952       X E +       PS       Heves       -0.056378       2.00167-017         0.7718       B +       0.7985       C +       0.0237       D +       0.8952       X E +       PS       Jstatus       -0.036678       2.00167-017       PS       Jstatus       -0.0366578       2.00167-017       PS       Jstatus       -0.036565       Astatus       -0.03656       Astatus       -0.03665       Astatus       -0.03656       Asta	0.9883       B +       0.8647       C +       0.6465       D +       0.9466       X +       Image: Constraint of the constrai															
1.0000       B + 1.0000       C + 0.0761       D + 0.0819       x + +       T         0.0464       B + 0.0896       C + 0.0460       D + 0.0812       x + +       T       FS Hogred -0.26324 -2.22045-016         0.7785       B + 0.9394       C + 0.1372       D + 0.0812       x + +       FS Hogred -0.26324 -2.22045-016         0.8725       B + 0.9394       C + 0.1372       D + 0.08030       x + +       FS Hogred -0.030228       2.08167-016         0.7718       B + 0.6608       C + 0.1277       D + 0.09328       x + +       FS Hogred -0.040256       4.8128-017         0.8057       B + 0.7535       C + 0.02954       D + 0.09328       x + +       FS Hogred -0.04256       4.85728-017         0.8356       B + 0.6291       C + 0.01813       D + 0.08358       x + +       C       0.161658       3.1332e-015         0.7385       B + 0.6291       C + 0.01813       D + 0.08358       x +       C       0.161658       3.1332e-015         0.7385       B + 0.6291       C + 0.01813       D + 0.08358       x +       C       0.161658       3.1332e-015         0.7385       B + 0.6808       C + 0.01581       D + 0.08318       x +       0.905003 -2.70096-015       D.904077-6.32766-016         0.7855       <	1.0000       B + 1.0000       C + 0.0761       D + 0.0819       XX + 1       F85_Hograd       -0.26324 - 2.2045-010         0.0864       B + 0.0966       C + 0.0761       D + 0.0812       XX + 1       F85_Hograd       -0.26324 - 2.2045-010         0.07685       B + 0.0934       C + 0.0327       D + 0.0912       XX + 1       F85_Hograd       -0.030282       2.08167-016         0.07785       B + 0.0505       C + 0.0237       D + 0.0952       XX + 1       F85_Hograd       -0.030282       2.08167-016         0.08057       B + 0.0505       C + 0.0237       D + 0.09552       XX + 1       F85_Hograd       -0.03276       2.08167-016         0.08057       B + 0.0505       C + 0.0234       D + 0.02358       XX + 1       0.041159       7.63276-017         0.08057       B + 0.05346       C + 0.0234       D + 0.0328       XX + 1       0.041159       7.63276-017         0.08056       B + 0.05346       C + 0.01813       D + 0.0533       XX + 1       0       0.041597-612596         0.07358       B + 0.0608       C + 0.01581       D + 0.06355       XX + 1       0       0.095103-7.0096-013         0.07458       B + 0.0608       C + 0.01581       D + 0.06355       XX + 1       0       0.040503-7.0296-015															
0.8464 B + 0.9868 C + 0.0450 D + 0.8512 x E + 0.7818 B + 0.7916 C + 0.1372 D + 0.8956 x E + 0.7518 B + 0.7916 C + 0.0227 D + 0.8956 x E + 0.7518 B + 0.7916 C + 0.0227 D + 0.8956 x E + 0.8067 B + 0.7936 C + 0.1977 D + 0.8956 x E + 0.8067 B + 0.7936 C + 0.1977 D + 0.8952 x E + 0.8056 B + 0.7365 C + 0.1813 D + 0.8238 x E + 0.8356 B + 0.7356 C + 0.1813 D + 0.8238 x E + 0.8356 B + 0.7356 C + 0.1951 D + 0.8328 x E + 0.8356 B + 0.5291 C + 0.1813 D + 0.8253 x E + 0.7358 B + 0.6690 C + 0.1591 D + 0.8355 x E + 0.7858 B + 0.6600 C + 0.1762 D + 0.8359 x E + 0.7858 B + 0.6600 C + 0.1762 D + 0.8359 x E + D 0.8669 B + 0.8600 C + 0.1762 D + 0.8359 x E + D 0.8669 B + 0.8600 C + 0.1762 D + 0.8359 x E + D 0.90580 - 2.7009e-015 0.7858 B + 0.6600 C + 0.1762 D + 0.8519 x E + D 0.90580 - 2.7009e-015 I D 0.90580 - 2.2766e-015 I D 0.90580 - 2.2766e-01	0.8464       B + 0.9868       C + 0.0460       D + 0.812       X + 1         0.7865       B + 0.9394       C + 0.0372       D + 0.9304       X + 1         0.7872       B + 0.7316       C + 0.0237       D + 0.9896       X + 1         0.7872       B + 0.7696       C + 0.0237       D + 0.9896       X + 1         0.7872       B + 0.7696       C + 0.0237       D + 0.9896       X + 1         0.7872       B + 0.7696       C + 0.0237       D + 0.9896       X + 1         0.8055       B + 0.7586       C + 0.0237       D + 0.9328       X + 1         0.8056       B + 0.7584       C + 0.0259       D + 0.9328       X + 1         0.8256       B + 0.7686       C + 0.1813       D + 0.9328       X + 1         0.8256       B + 0.6690       C + 0.0580       D + 0.9328       X + 1         0.8257       B + 0.6690       C + 0.0580       D + 0.9328       X + 1         0.77358       B + 0.6690       C + 0.0580       D + 0.9328       X + 1         0.7859       B + 0.6690       C + 0.0580       D + 0.9328       X + 1         0.7859       B + 0.6690       C + 0.1762       D + 0.93245       X = 1         D 0.990172       Free       D 0.9901															
0.7885       B + 0.9334       C + 0.1372       D + 0.0836       xE +         0.8722       B + 0.9316       C + 0.1237       D + 0.0836       xE +         0.8722       B + 0.9316       C + 0.1237       D + 0.0836       xE +         0.8067       B + 0.6695       C + 0.1237       D + 0.9358       xE +         0.8067       B + 0.5785       C + 0.2034       D + 0.9328       xE +         0.8067       B + 0.5745       C + 0.0235       D + 0.9328       xE +         0.8355       B + 0.6291       C + 0.01813       D + 0.9328       xE +         0.8356       B + 0.6291       C + 0.01813       D + 0.93258       xE +         0.8356       B + 0.6291       C + 0.01813       D + 0.93258       xE +         0.8356       B + 0.6291       C + 0.01813       D + 0.93258       xE +         0.8356       B + 0.6690       C + 0.01812       D + 0.93258       xE +         0.8356       B + 0.6690       C + 0.01762       D + 0.03218       xE +         0.7858       B + 0.6690       C + 0.01762       D + 0.0319       xE +         10.349037       J 0.991272       A.2398e-013         bounds       IFS_Fest       Ifree       IFS_Fest       Ifree </td <td>0.7885       B + 0.9334       C + 0.1372       D + 0.0805       xE +         0.7895       B + 0.9334       C + 0.1272       D + 0.9395       xE +         0.7815       B + 0.6668       C + 0.1277       D + 0.9395       xE +         0.7815       B + 0.7358       C + 0.1277       D + 0.9395       xE +         0.7815       B + 0.7358       C + 0.1277       D + 0.9355       xE +         0.8055       B + 0.7358       C + 0.1294       D + 0.9238       xE +         0.8055       B + 0.7356       C + 0.1934       D + 0.9238       xE +         0.8256       B + 0.7326       C + 0.1813       D + 0.8237       xE +         0.8256       B + 0.7365       C + 0.0591       D + 0.8237       xE +         0.7412       B + 0.6800       C + 0.0590       D + 0.8235       xE +         0.7358       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7412       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7658       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7658       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.9904727-6.5298e-015       D + 0.8245       xE +<!--</td--><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>Π</td><td></td><td></td><td></td></td>	0.7885       B + 0.9334       C + 0.1372       D + 0.0805       xE +         0.7895       B + 0.9334       C + 0.1272       D + 0.9395       xE +         0.7815       B + 0.6668       C + 0.1277       D + 0.9395       xE +         0.7815       B + 0.7358       C + 0.1277       D + 0.9395       xE +         0.7815       B + 0.7358       C + 0.1277       D + 0.9355       xE +         0.8055       B + 0.7358       C + 0.1294       D + 0.9238       xE +         0.8055       B + 0.7356       C + 0.1934       D + 0.9238       xE +         0.8256       B + 0.7326       C + 0.1813       D + 0.8237       xE +         0.8256       B + 0.7365       C + 0.0591       D + 0.8237       xE +         0.7412       B + 0.6800       C + 0.0590       D + 0.8235       xE +         0.7358       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7412       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7658       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.7658       B + 0.6800       C + 0.0590       D + 0.8245       xE +         0.9904727-6.5298e-015       D + 0.8245       xE + </td <td></td> <td>Π</td> <td></td> <td></td> <td></td>												Π			
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0.7318       B + 0.6669       C + 0.1477       D + 0.0325       x E +         0.8057       B + 0.7558       C + 0.1204       D + 0.0228       x E +         0.8057       B + 0.7558       C + 0.1204       D + 0.0228       x E +         0.8057       B + 0.7565       C + 0.1203       D + 0.0228       x E +         0.8355       B + 0.7565       C + 0.1813       D + 0.0237 x E +       C - 0.016264       9.6727 x E +         0.8355       B + 0.6691       C + 0.0518       D + 0.0235       x E +       C - 0.161658       3.1332e-017         0.7412       B + 0.6693       C + 0.0518       D + 0.0235       x E +       C - 0.161658       3.1332e-013         0.7695       B + 0.6690       C + 0.0518       D + 0.0335       x E +       C - 0.161658       3.1332e-013         0.7858       B + 0.6690       C + 0.0578       D + 0.0335       x E +       N - 0.90560-015         0.7858       B + 0.6690       C + 0.01762       D + 0.0319       x E +       N - 0.44903'' - 6.2398e-013         bunds       IFS_Fest       free       IFS_Fest       If ee       IFS_Fest       If ee         IFS_Fest       free       IFS_Fest       If ee       IFS_Fest       If ee       IFS_Fest	0.7333       B + 0.6698       C + 0.1377       D + 0.9392       XE +         0.8055       B + 0.7358       C + 0.1294       D + 0.9392       XE +         0.8055       B + 0.7358       C + 0.1294       D + 0.9238       XE +         0.8055       B + 0.7358       C + 0.0954       D + 0.9238       XE +         0.8055       B + 0.7356       C + 0.0954       D + 0.9238       XE +         0.8250       B + 0.7326       C + 0.1813       D + 0.8237       XE +         0.8355       B + 0.6591       C + 0.0591       D + 0.8031       XE +       0.90500 -2.70096-015         0.3254       B + 0.6690       C + 0.0590       D + 0.8245       XE +       N 0.9050-2.70096-015         0.7858       B + 0.6690       C + 0.0590       D + 0.8245       XE +       N 0.9050-2.70096-015         0.8680       B + 0.6690       C + 0.0590       D + 0.8245       XE +       I 0.90591272-6.2398e-015         0.9060       C + 0.01762       D + 0.8245       XE +       I 0.90591272-6.2398e-015         D.90600       C + 0.01762       D + 0.8245       XE +       I 0.904077-6.2398e-015         D.90600       C + 0.01762       D + 0.8245       XE +       I 0.904077-6.2398e-015         D.906000															
0.7310       B + 0.7326       C + 0.1417       D + 0.0332       X +         0.8067       B + 0.7336       C + 0.2946       D + 0.0332       X +         0.8067       B + 0.7356       C + 0.0954       D + 0.0338       X +         0.8057       B + 0.7356       C + 0.0954       D + 0.0338       X +         0.8256       B + 0.7365       C + 0.1813       D + 0.0237       X +         0.8356       B + 0.5294       C + 0.1813       D + 0.02517       X +         0.7316       B + 0.5294       C + 0.0591       D + 0.02537       X +         0.7326       B + 0.5294       C + 0.0591       D + 0.02537       X +         0.7326       B + 0.5294       C + 0.0591       D + 0.0353       X +         0.7326       B + 0.6600       C + 0.0591       D + 0.0355       X +         0.7826       B + 0.6600       C + 0.1762       D + 0.03519       X +         Nounds       TPS Pest       free       Free       J       0.098127       -6.23766e-015         PS Kombsti free       Free       Free       J       Jopelanci       D       D         PS Kombsti free       Free       Free       D       D       D       D	0: 2026       B + 0       0: 2036       C + 0       0: 2036       XE +         0: 2036       B + 0       0: 2036       XE +       0: 2036       XE +         0: 2036       B + 0       0: 2036       XE +       0: 2036       XE +         0: 2036       B + 0       0: 2036       XE +       0: 2036       ASSA         0: 2036       B + 0       0: 2036       XE +       0: 2036       ASSA         0: 2036       B + 0       0: 2036       XE +       0: 0: 2036       ASSA         0: 2036       B + 0       0: 2036       XE +       0: 0: 0: 0: 0: 0: 0: 0: 0: 0: 0: 0: 0: 0		0.87	22	B +	0.791	6 C	+ 0	.0237	D +	0.8956	×E +				
0.00077       B + 0.05946       C + 0.0058       D + 0.0328       AE + 0.0328       AE + 0.0328       AE + 0.0328       AE + 0.01256       A87328-017       AE + 0.01256       AE	0.3000       B + 0.7345       C + 0.0394       D + 0.3226       A.E +         0.3000       B + 0.7345       C + 0.0394       D + 0.3226       A.E +         0.3256       B + 0.7365       C + 0.1813       D + 0.6217       XE +         0.3256       B + 0.7365       C + 0.1813       D + 0.6217       XE +         0.3256       B + 0.6591       C + 0.1813       D + 0.6031       XE +         0.3256       B + 0.6590       C + 0.0591       D + 0.6031       XE +         0.3256       B + 0.6690       C + 0.0590       D + 0.63515       XE +         0.0656       B + 0.6690       C + 0.0590       D + 0.63519       XE +         0.0656       B + 0.6690       C + 0.1762       D + 0.63519       XE +         1.0.449037       -6.2398e-015       Jounds       Jounds -J.7677e-015         1.2.766e-015       Jounds       Jounds -J.7677e-015       Jounds -J.7677e-015         1.2.8.76       free       LPS Fear       C - 0.019172 - 6.2398e-015         1.2.9       Jounds -J.7677e-015       Jounds -J.7677e-015       Jounds -J.7677e-015         1.2.9       Jounds -J.7677e-015       Jounds -J.7677e-015       Jounds -J.7677e-015         1.2.9       Joungametrix weecharged       C - 0.01762		0.75	18	B +	0.606	9 C	+ 0	.1477	D +	0.8952	xE +	EPS_			
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0.8258       B +       0.7365       C +       0.813       D +       0.8237       xE +         0.8256       B +       0.6251       D +       0.8237       xE +       D +       0.8236       xE +         0.7412       B +       0.6031       D +       0.8031       xE +       D +       0.90500 - 2.30090e-015         0.6358       B +       0.6690       C +       0.0590       D +       0.8355       xE +         0.7858       B +       0.6690       C +       0.0500       D +       0.8245       xE +         0.6663       B +       0.6600       C +       0.1762       D +       0.8245       xE +         1.0.449037       -6.2396e-015       D       D       0.905127       -6.22766e-015         1.785       Pejer       free       FPS_Fejer       free       FPS_Fejer       Tree         FPS_Fejer       free       FPS_Fejer       free       FPS_Fejer       Tree       Free         IDALMARY       free       FPS_Fejer       If end       Sats       Matrix       Solutions       Objective       MIP search         IDALMARY       free       IIS       III       Sats       Matrix       Solutions	0.8256       B + 0.7365       C + 0.1813       D + 0.8237       x X + 0.8237       x X + 0.06237       x X + 0.06031       x X + 0.06031       x X + 0.06037       x X + 0.06037       x X + 0.06037       x X + 0.06031       x X + 0.00631       x		0.80	67	B +	0.594	6 C	+ 0	.0954	D +	0.8328	×E +	EI EI			
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0.7412         B +         0.6540         C +         0.0351         D +         0.0801         xE +           0.8394         B +         0.6803         C +         0.0588         D +         0.8355         xE +           0.7612         B +         0.6803         C +         0.0598         D +         0.8355         xE +           0.7625         B +         0.6800         C +         0.0300         D +         0.8355         xE +           0.6869         D +         0.8319         XE +         H         0.146000         -1.76677e-015           bounds         EPS / Feir         free         F         J         0.0981272         -6.2398e-015           Type here:         EPS / Feir         free         F         Type here:         Type here:         Type here:           PS Kombst free         EPS / Feir         free         E         E         E         E         E         E	0.7412       B + 0.6548       C + 0.0554       D + 0.0311       xE + 0.0541       D + 0.0311       xE + 0.0541       D + 0.0355       xE + 0.0556       D + 0.0355       xE + 0.0567       xE + 0				B +	0.629	1 C	+ 0	1813	D +	0.8553	XE +				
0.8394         B +         0.8003         C +         0.0398         D +         0.8355         xE +           0.7855         B +         0.6890         C +         0.0500         D +         0.8355         xE +           0.7855         B +         0.6890         C +         0.1500         D +         0.8286         1.78677-013           0.7855         B +         0.6890         C +         0.1762         D +         0.819         xE +           10.48037         -6.22366-013         3         0.0901272-6.22766-013           PS Past         free         Image: State Stat	0.8354         B +         0.6003         C +         0.0358         D +         0.8355         X +         H         0.14600         1.78677-0.13           0.7655         B +         0.6890         C +         0.0500         D +         0.8355         X +         H         0.14600         1.78677-0.13           0.8669         B +         0.6890         C +         0.0500         D +         0.8355         X +         H         0.14600         1.78677-0.13         0.090127         0.23980-013           0.8669         B +         0.8690         C +         0.01762         D +         0.8519         X +         I         0.149037         -6.23980-013           Dounds         IFS_Fejet         free         IFS_Fejet         Ifee															
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(CSIZMADIA G. 2009.)

The aim of the model based on the such determined development function is to find those factors for the 3 counties, to where it would be the most efficient to concentrate the development efforts on. To this aim, the following model finds those values of the development function that requires the smallest investment (in terms of deviation from the current values) but brings the value of the development function level with the national average GDP level used as a reference value.

 $\min(x_A - x^{current}_A)^2 + \dots + (x_J - x^{current}_J)^2 \quad \text{(looking for the solution}$ with minimal deviation)  $Ax_A + \dots + Jx_J = 1,78 \quad \text{(for which the approximation reaches the average}$ GDP)

$X_A$	$\geq x_A^{current}$
$X_{I}$	$\sum x_{I}^{current}$

Their might be some explanations due on the assumptions that the aim coefficient variables are at least as large as the current levels. This is because of the nature of operations research models applied for economical models, i.e. without that the models might "sell" some factors in order to free up resources for factors with higher margins (a representative example might be the production of a factory: without such constraints we might get solutions suggesting to buy back less cost effective products, disassemble them, then reassemble into models which are more cost efficient and sell them again. This would also mean the obvious over-optimization of the model).

Solving the model, I got the following results for the counties of the Trans-Danubian region.

Baranya	Current normalized	Required by	Difference
	value	solution	
В	0,846	0,846	0,00%
С	0,897	1,429	59,37%
D	0,046	0,046	0,00%
E	0,851	0,974	14,48%
Н	0,496	0,661	33,31%
1	0,895	0,895	0,00%
J	0,485	0,749	54,40%

Table 1. Fields to invest inprovment on according to the model.

Somogy	Current	Solution	Difference
В	0,789	0,789	0,00%
С	0,933	1,548	65,80%
D	0,137	0,137	0,00%
E	0,803	0,968	20,57%
Н	0,107	0,107	0,00%
	0,854	0,854	0,00%
J	0,689	1,183	71,86%
Tolna	Current	Solution	Difference
В	0,872	0,872	0,00%
			- )
С	0,792	1,281	61,87%
C D	,	1,281 0,024	,
	0,792	,	61,87%
D	0,792 0,024	0,024	61,87% 0,00%
D E	0,792 0,024 0,896	0,024 1,188	61,87% 0,00% 32,67%
D E	0,792 0,024 0,896 0,062	0,024 1,188 0,062	61,87% 0,00% 32,67% 0,00%

In case of Baranya, the model emphasizes the importance of improving the willingness to start new enterprises (C), the improvement of transportation infrastructure (J), and with less importance the spending on research and development (H) and slightly the compensation situation of physical workers.

For Somogy, one of the main factors of importance would also be the willingness to start new enterprises (C), but even more importantly the improvement of transportation infrastructure (J), and similarly, the compensation situation of physical workers (E) is also part of the solution.

The results for Tolna are similar to Somogy, although the model gave a small nonzero priority to the development of communication networks as well.

# 5. Summary

I have presented several mathematical models, with which the efficiency of the efforts aiming to ease the disadvantaged situation of the region can be increased. With the help of several concrete numerical calculations, I have drawn conclusions and provided development preferences. The robustness and flexibility of the model provides quick reassessment in case of unexpected events.

The main results can be summarized as follows:

- The thesis summarizes the current economical situation and expectations of the Trans-Danubian Region, provides an extensive introduction to the generally applicable mathematical methods and models, including the disciplines of statistics, optimization and data mining.
- The conclusions are backed with extensive data collection efforts.
- The thesis provides models for quick decision support in case of unexpected events.
- It analysis problems related to project management and human resources and applies resource allocation models for their optimal solutions.
- It develops and introduces risk analysis and recognition methods.
- It develops and provides numerical results and their analysis for the calculation of optimal development plans for the Trans-Danubian region.

The development of the region would not be possible without innovation. There is a need for new ideas, new activities, new products and organizations. There is also a need for economyorganization type innovation; product innovation; social-political innovation.

It must be achieved, that together with the improvement of the communication systems, new innovations must develop both in time and space as quickly as possible. An enterprise friendly environment must be formed (supporting banking system, enterprise capital, risk capital) and enterprise friendly economical politics is necessary. The means of this could be:

- science parks (e.g. innovation centers in Pécs)
- technology transfer bases
- incubation houses
- industrial parks

although the environmental conditions should also be met:

- human resource
- market view

Together with the developments based on the regions internal resources aiming to provide for the local market needs (especially due to needs arising with the changes in agriculture) efforts must be made to allocate a larger fraction of nation wide resources and to integrate more into the international division of labor.

Several presented models will require more detailed analysis and significant data collection effort to make then viable for real life applications, tough based on the presented results their application is very promising. For some models, the application would require government level cooperation (like in the case of the presented risk analysis models).

The fundamental idea of applications is based on the simple customizability and re-calculability of the methods. In case of unforeseeable, unexpected economical situations, natural disasters, or any other, region wide events, the optimal allocation (or reallocation) of available resources the negative effects can be minimized.

The results of the thesis are based on several already published or submitted papers. The presented models are based on well known mathematical disciplines; however their applications to the aimed problems are new.

The developed models range from one aiming to assist the development of the economy, through decision support system and human management allocation models to methods of risk analysis. Concrete applications are shown on the example of the Trans-Danubian region.

A significant part of the data used for the analysis in the thesis has never been collected and made available in this form before.

Thanks to the relatively simple applied technology the models do not require special computational hardware background. Connecting the model with the already existing geo-informatics systems promise flexible models that are easily populated with up to date data. Several model can be applied and localized based on widespread readily available software, or at least the applicability can be assessed using no more demanding computational background that is provided by Excel's statistical and solver functions. The thus developed models can then be applied to large scale, real life applications.

### **6.** Publications

### Published literature related to PhD topic

 CSIZMADIA G. 2007: Matematikai modellezési lehetőségek a Dél-Dunántúli és a Dél-Alföldi régió gazdaság-elemzése és gyors döntés-előkészítéshez rendkívüli esemény esetén. Szakmai Tudományos Közlemények. Az MK Katonai Biztonsági Hivatal Tudományos Kutatóhely és az MH Geoinformációs Szolgálat Kiadványa. Budapest, ISSN 1586-099X, CD kiadvány, 4 p.

- CSIZMADIA G. 2008: The Application of Statistical Forecasting Models to Reveal the Main Risk Factors in The Future of the Southern-Trans-Danubian Region. – Tradecraft Review, 2008 Special Issue, Budapest, ISSN 1785-1181, pp. 84-90.
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- 5. CSIZMADIA G. 2009. *Risk analysis in a changing word.* Tradecraft Review 2009. Special Issue, pp. 92-96.
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#### **Conference presentations related to PhD topic**

 CSIZMADIA G. 2004: A Dél-Dunántúli régió gazdasági, társadalmi terére ható geográfiai tényezők. In: Barton G. – Dormány G – Rakonczai J. (szerk.): Földrajzi kutatások 2004.. Absztrakt kötete ISBN 963 482 686 5, Szeged, 2004. p.39.

- CSIZMADIA G. 2005: A Dél-Dunántúli Régió versenyképességét befolyásoló geográfiai tényezők. In: Pap N. – Végh A. (szerk.).- A Kárpát-medence politikai földrajza. PTE TTK FI KMBTK, Pécs, 2005. p. 2.
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#### **Other publications**

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- 18. CSIZMADIA G. 2009. *Kockázatelemzés támogatás egy változó világban.* KBH Szakmai Szemle 2009. 2. szám, pp. 204-208.